Cosmological inflation in f(R,G) gravity

Mariafelicia De Laurentis

Institute for Theoretical Physics
Goethe University
Frankfurt

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Summary

- Extending F(R)-gravity and the role of Gauss-Bonnet invariant
- Field equations of F(R,G)-gravity and solutions
- F(R,G) double inflation
- F(R,G) power-law inflation
- Conclusions

Extending F(R)-gravity

- F(R) gravity is the simplest extension of General Relativity but it does not contains all the curvature invariants.

- in order to consider the whole curvature budget we need also the Ricci tensor $R_{\mu\nu}$ the Riemann tensor $R^{\mu}{}_{\nu\gamma\delta}$, and the Weyl tensor $C^{\mu}{}_{\nu\gamma\delta}$.

- However due to the algebraic relation among this geometric objects given by the Gauss Bonnet invariant, we need just R and G

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$$

From the quantum field theory point of view G plays a fundamental role in quantizing fourth order gravity

- It regularizes the theories (N. H. Barth and S. M. Christensen, PRD 28, 8 1983)
- It plays a main role in defining the trace anomaly (S. Capozziello, M. De Laurentis, Phys. Rep. 509, 167 ,2011)

F(R,G) gravity

A general action is

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}F(R,\mathcal{G}) + \mathcal{L}_m\right]$$

The variation provides the following field equations

$$\begin{aligned} G_{\mu\nu} = \frac{1}{F_R}\Big[& \nabla_\mu\nabla_\nu F_R - g_{\mu\nu}\Box F_R + 2R\nabla_\mu\nabla_\nu F_G \\ & - 2g_{\mu\nu}R\Box F_G - 4R_\mu{}^\lambda\nabla_\lambda\nabla_\nu F_G - 4R_\nu{}^\lambda\nabla_\lambda\nabla_\mu F_G \\ & + 4R_{\mu\nu}\Box F_G + 4g_{\mu\nu}R^{\alpha\beta}\nabla_\alpha\nabla_\beta F_G + 4R_{\mu\alpha\beta\nu}\nabla^\alpha\nabla^\beta F_G \\ & - \frac{1}{2}g_{\mu\nu}(RF_R + \mathcal{G}F_\mathcal{G} - F(R,\mathcal{G}))\Big]. \end{aligned}$$

G. Cognola, E. Elizalde, S. Nojiri, S. Odintsov and S. Zerbini, Phys. Rev. D 75, 086002 (2007)
M. De Laurentis and A. J. Lopez-Revelles Int. J. Geom. Meth. Mod. Phys. 11, 1450082 (2014)
R. Myrzakulov, L. Sebastiani, S. Zerbini Int. Jour. of Mod. Phys. D, 22 1330017 (2013).

F(R,G) gravity

Let us focus on the trace equation

$$-2F(R,\mathcal{G}) + RF_R + 3\nabla^2 F_R$$
$$+2\mathcal{G}F_{\mathcal{G}} + 2R\nabla^2 F_{\mathcal{G}} - 4R_{\rho\sigma}\nabla^\rho\nabla^\sigma F_{\mathcal{G}} = 0.$$

We can recast as follow

$$3\left[\Box F_R + V_R\right] + R\left[\Box F_{\mathcal{G}} + W_{\mathcal{G}}\right] = 0,$$

where we can distinguish two different potential,

$$V_R = \frac{\partial V}{\partial R} = \frac{1}{3}\left[RF_R - 2F(R,\mathcal{G})\right]$$

$$W_{\mathcal{G}} = \frac{\partial W}{\partial \mathcal{G}} = 2\frac{\mathcal{G}}{R}F_{\mathcal{G}}$$

M. De laurentis Mod.Phys.Lett. A30 12, 1550069 (2015)

M. De Laurentis, M. Paolella, S. Capozziello PRD 91, 083531 (2015)

Reducing to a canonical point-like Lagrangian

The point-like canonical Lagrangian

$$\mathcal{L}(a, \dot{a}, R, \dot{R}, \mathcal{G}, \dot{\mathcal{G}})$$

$$\mathcal{Q} \equiv \{a, R, \mathcal{G}\}$$ configuration space

$$\mathcal{TQ} \equiv \{a, \dot{a}, R, \dot{R}, \mathcal{G}, \dot{\mathcal{G}}\}$$ tangent space

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right), \qquad \mathcal{G} = 24\left(\frac{\ddot{a}\dot{a}^2}{a^3}\right)$$

Euler-Lagrange equations are

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{a}} = \frac{\partial \mathcal{L}}{\partial a}, \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{R}} = \frac{\partial \mathcal{L}}{\partial R}, \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\mathcal{G}}} = \frac{\partial \mathcal{L}}{\partial \mathcal{G}}$$

$$E_{\mathcal{L}} = \frac{\partial \mathcal{L}}{\partial \dot{a}}\dot{a} + \frac{\partial \mathcal{L}}{\partial \dot{R}}\dot{R} + \frac{\partial \mathcal{L}}{\partial \dot{\mathcal{G}}}\dot{\mathcal{G}} - \mathcal{L} = 0$$

Reducing to a canonical point-like Lagrangian

Selecting the suitable Lagrange multiplier..

$$\mathcal{A} = 2\pi^2 \int dt\, a^3 \left\{ F(R,\mathcal{G}) - \lambda_1 \left[R + 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) \right] - \lambda_2 \left[\mathcal{G} - 24 \left(\frac{\ddot{a}\dot{a}^2}{a^3} \right) \right] \right\}$$

Lagrange multipliers

$$\lambda_1 = \frac{\partial F(R,\mathcal{G})}{\partial R} \qquad \lambda_2 = \frac{\partial F(R,\mathcal{G})}{\partial \mathcal{G}}$$

$$\mathcal{A} = 2\pi^2 \int dt \left\{ a^3 F(R,\mathcal{G}) - a^3 \frac{\partial F(R,\mathcal{G})}{\partial R} \left[R + 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) \right] - a^3 \frac{\partial F(R,\mathcal{G})}{\partial \mathcal{G}} \left[\mathcal{G} - 24 \left(\frac{\ddot{a}\dot{a}^2}{a^3} \right) \right] \right\}$$

$$\mathcal{L} \;=\; 6a\dot{a}^2 F_R + 6a^2\dot{a}\dot{F}_R - 8\dot{a}^3\dot{F}_\mathcal{G} + a^3 \left[F(R,\mathcal{G}) - R\, F_R - \mathcal{G} F_\mathcal{G} \right]$$

S. Capozziello, M. De Laurentis and S. D. Odintsov, Mod. Phys. Lett. A 29, 1450164 (2014).

F(R,G) double Inflation

$$\dot{H} = \frac{1}{2F_R + 8H\dot{F}_{\mathcal{G}}}\left[H\dot{F}_R - \ddot{F}_R + 4H^3\dot{F}_{\mathcal{G}} - 4H^2\ddot{F}_{\mathcal{G}}\right]$$

$$H^2 = \frac{1}{6\,F_R + 24H\dot{F}_{\mathcal{G}}}\left[F_R R - F(R,\mathcal{G}) - 6H\dot{F}_R + \mathcal{G}F_{\mathcal{G}}\right]$$

For inflationary scenario

$$|\frac{\dot{H}}{H^2}| \ll 1 \qquad |\frac{\ddot{H}}{H\,\dot{H}}| \ll 1$$

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = -\frac{\ddot{H}}{2\,H\,\dot{H}}$$

must be small during inflation

Moreover, ε > 0 in order to have H < 0 , and, since the acceleration is expressed as

$$\Rightarrow \quad \frac{\ddot{a}}{a} = \dot{H} + H^2$$

F(R,G) double inflation

Let us choose, the Lagrangian

$$F(R,\mathcal{G}) = R + \alpha R^2 + \beta \mathcal{G}^2$$

$$\mathcal{L} = \underbrace{K(q_i, \dot{q}_j)}_{\text{kinetic energy}} - \underbrace{U(q_i)}_{\text{potential energy}}$$

kinetic energy potential energy

considering the Lagrangian density, i.e. L =a^{3}L, it is

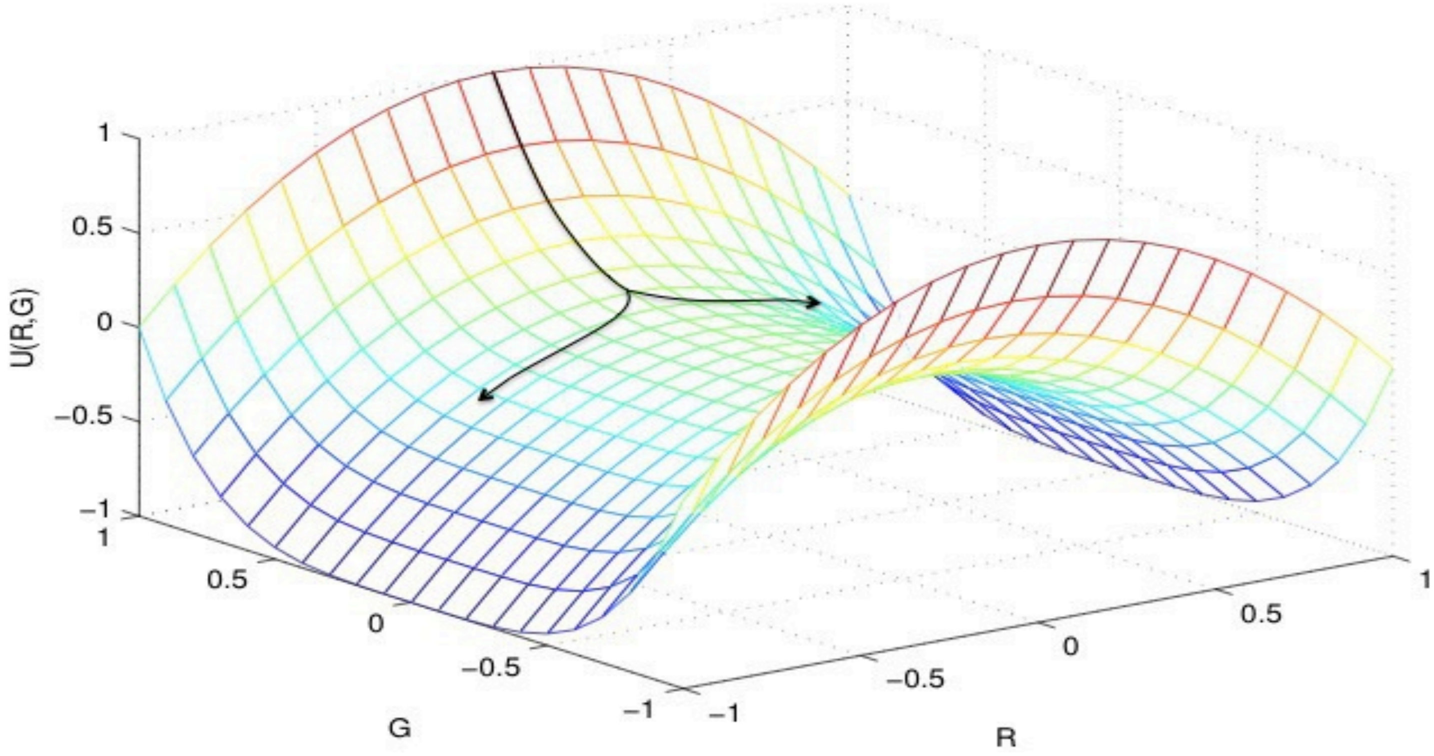
$$K(a, \dot{a}, R, \dot{R}, \mathcal{G}, \dot{\mathcal{G}}) = 6\left(\frac{\dot{a}}{a}\right)^2 F_R + 6\left(\frac{\dot{a}}{a}\right)\dot{F}_R - 8\left(\frac{\dot{a}}{a}\right)^3 \dot{F}_{\mathcal{G}},$$

$$U(R, \mathcal{G}) = -\left[F(R, \mathcal{G}) - RF_R - \mathcal{G}F_{\mathcal{G}}\right].$$

$$L = \overbrace{6\left(\frac{\dot{a}}{a}\right)^2 (2\alpha R + 1) + 12\alpha\left(\frac{\dot{a}}{a}\right)\dot{R} - 16\beta\left(\frac{\dot{a}}{a}\right)^3 \dot{\mathcal{G}}}^{\text{kinetic energy}} - \underbrace{[\beta\mathcal{G}^2 + \alpha R^2]}_{\text{potential energy}}$$

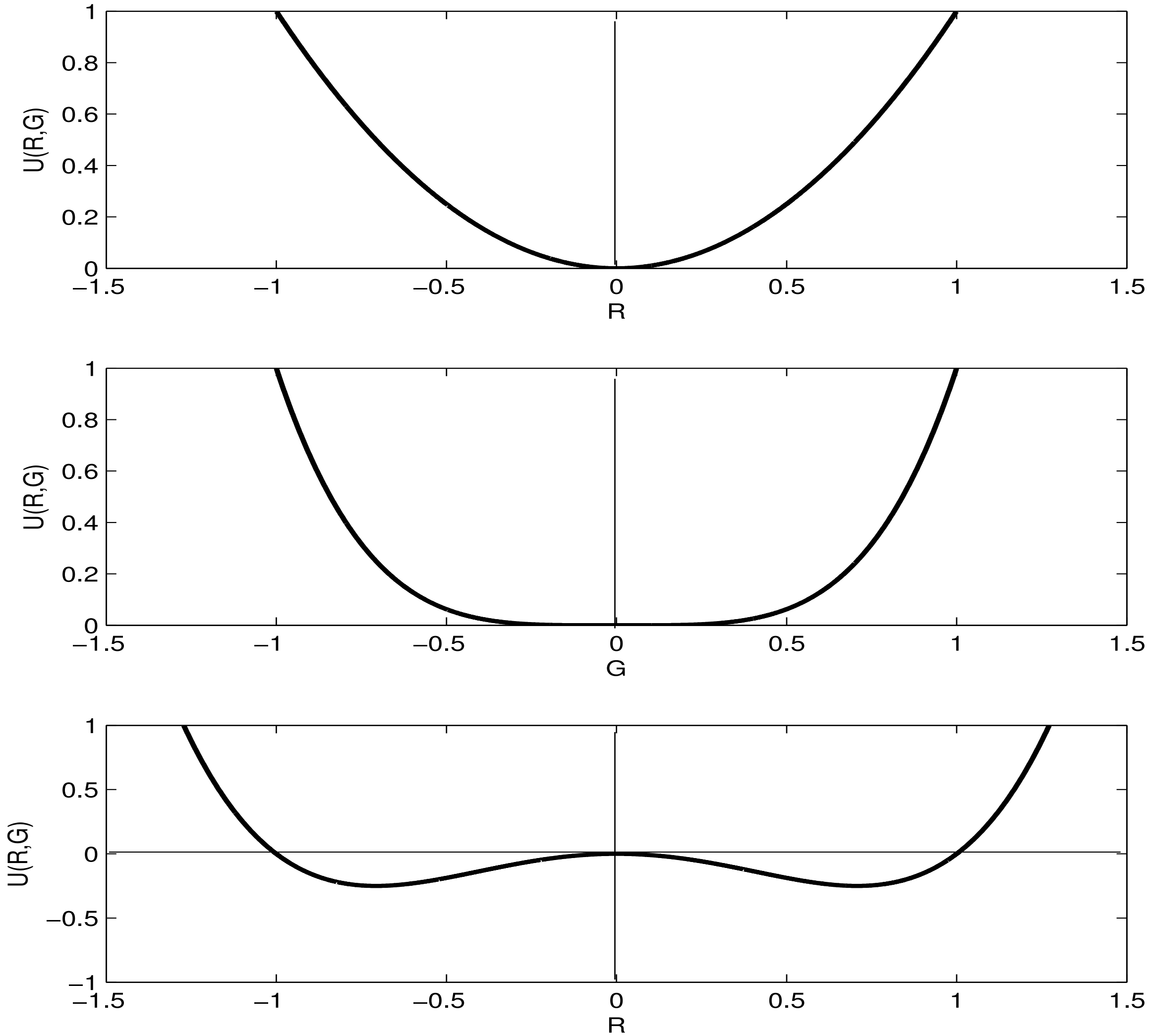
F(R,G) double inflation

Plot of U(R,G)= $\alpha R^2 + \beta G^2$



F(R,G) double inflation

plot of $U(R,G) = \alpha R^2 + \beta G^2 = \alpha R^2 + \beta R^4$



F(R,G) double inflation

Friedmann equations become

$$
\begin{aligned}
&12\alpha H\ddot{H} + H^2 + 36\alpha H^2\dot{H} + 288\beta H^4\dot{H}^2 \\
&+192\beta H^5\ddot{H} + 576\beta H^6\dot{H} - 96\beta H^8 - 6\alpha\dot{H}^2 = 0
\end{aligned}
$$

$$
\begin{aligned}
&576\beta H^2\dot{H}^3 + 768\beta H^3\dot{H}\ddot{H} + \beta H^4\left(1728\dot{H}^2 + 96\dddot{H}\right) \\
&+288\beta H^5\ddot{H} - 384\beta H^6\dot{H}^2 + \\
&+18\alpha H\ddot{H} + 24\alpha\dot{H}^2 + 6\alpha\dddot{H} + \dot{H} = 0
\end{aligned}
$$

we shall find approximate solutions of above equations in various regimes

$$H^2 + 6\alpha\left(2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2\right) + 96\beta H^4\left(3\dot{H}^2 + 2H\ddot{H} + 6H^2\dot{H} - H^4\right) = 0$$

$6\alpha >> 96\beta H^4$ ← Two regimes → $96\beta H^4 >> 6\alpha$

Dominates R

Dominates G

F(R,G) double inflation

$6\alpha >> 96\beta H^4$

$$H^2 + 6\alpha\left(2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2\right) \cong 0$$

solution for the scale factor is

$$a(t) \sim \exp\left[\frac{t}{\sqrt{6\alpha}}\right]$$

$$m_R^2 = \frac{1}{6\alpha}$$ Scalaron mass

A. A. Starobinsky, Phys. Lett. B 117, 175 (1982).

On the other hand, we can consider the regime

$96\beta H^4 >> 6\alpha$

$$H^2 + 96\beta H^4\left(3\dot{H}^2 + 2H\ddot{H} + 6H^2\dot{H} - H^4\right) \cong 0$$

Inflation is recovered for $H^6 \sim \frac{1}{96\beta}$, $\quad a(t) \sim \exp\left[\frac{t}{\sqrt[6]{96\beta}}\right].$

$$m_{\mathcal{G}}^2 = \frac{1}{2\sqrt[3]{12\beta}}$$ Gauss Bonnet scalaron mass

M. De Laurentis, et al.PRD 91, 083531 (2015)

Power law inflation from Noether symmetries

We can use Noether symmetries to find solutions. A Noether symmetry exists if the condition

$$L_X \mathcal{L} = 0 \quad \Longrightarrow \quad X\mathcal{L} = 0$$

Lie derivative

Noether vector

the contraction of the Noether vector defined on the tangent space

$$\mathcal{TQ} = \{q_i, \dot{q}_i\} = \{a, \dot{a}, R, \dot{R}, \mathcal{G}, \dot{\mathcal{G}}\}$$

of the Lagrangian $\mathcal{L}(q_i, \dot{q}_i) = \mathcal{L}(a, \dot{a}, R, \dot{R}, \mathcal{G}, \dot{\mathcal{G}})$

with the Cartan one-form, generically define $\theta_{\mathcal{L}} \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}_i} dq^i$

Noether charge $i_X \theta_{\mathcal{L}} = \Sigma_0$

S. Capozziello, M. De Laurentis Int J. Geom. Meth. Mod. Phys. 11, 1460004 (2014).
S.Capozziello, M. De Laurentis, R. Myrzakulov, aInt. J. Geom. Methods Mod. Phys. DOI: 10.1142/S0219887815500954 rXiv:1412.1471 (2014).
S.Capozziello, M. De Laurentis, R. Myrzakulov, Int. J. Geom. Methods Mod. Phys. 12, 1550065 (2015)

Power law inflation

A possible choice for the Lagrangian $F(R,\mathcal{G}) = F_0 R^n G^{1-n}$

the point-like Lagrangian becomes, choosing the simplest non-trivial case n = 2,

$$\mathcal{L} \; = \; \frac{4\,F_0\,\dot{a}}{\mathcal{G}}\left[3\,a\,\dot{a}\,R + 3\,a\,\dot{R} - 3\,a^2\,\dot{\mathcal{G}}\frac{R}{\mathcal{G}} + 4\,\dot{a}^2\,\dot{R}\frac{R}{\mathcal{G}} - 4\,\dot{a}^2\,\dot{\mathcal{G}}\left(\frac{R}{\mathcal{G}}\right)^2\right]$$

Power law solutions are found for

$n = 2$ and $s = 3$ ⟵ $a(t) = t^s$ ⟶ $n = \frac{3}{4}$ and $s = \frac{1}{2}$

General conditions between the exponents n and s are

$$n = 0 \qquad n = \frac{1+s}{2} \qquad n = -2s + \frac{1}{1 + 2s(s-1)}$$

Power law inflation

To obtain inflation we rewrite the field equation in terms of n and s

$$\dot{H} = -\frac{s(n-1)\left[n(6s-4)-3s(s+1)+4\right]}{\left[s(s-5)+2n(2s-1)+2\right]t^2},$$

$$H^2 = -\frac{2s^2(s-1)(n-1)}{\left[s(s-5)+2n(2s-1)+2\right]t^2},$$

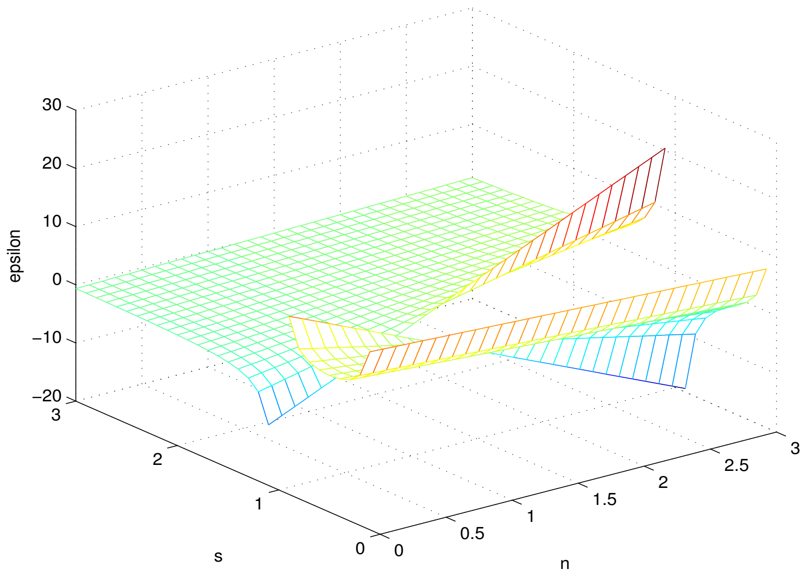
A condition for inflation is

$$|\frac{\dot{H}}{H^2}| = |\frac{2s(n-1)-2(s-1)+s(s-1)}{2s(s-1)}| \ll 1.$$

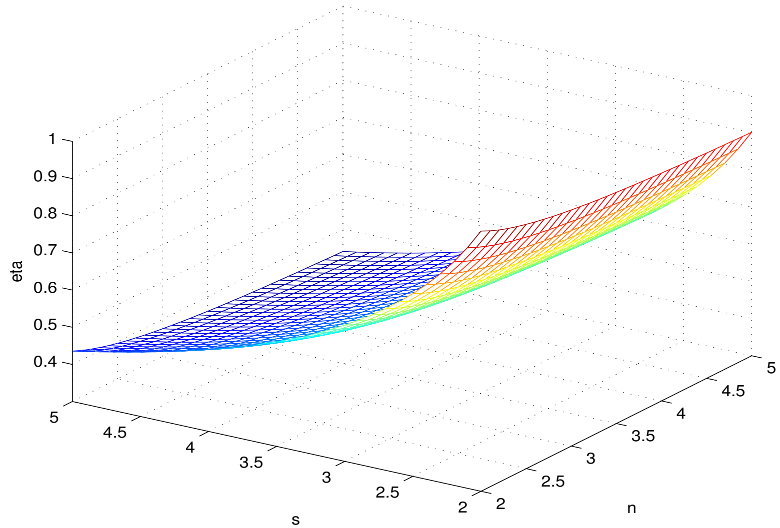
Power law inflation

The slow-roll conditions are

$$\epsilon = \frac{-2s(n-1) + 2(s-1) - s(s-1)}{2s(s-1)} \ll 1,$$



$$\eta = \frac{1}{\sqrt{2}}\sqrt{\frac{s^2(s-1)(n-1)}{s(s-5) + n(4n-2) + 2}} \ll 1$$



Power law inflation

The spectral index ns and the tensor-to-scalar ratio are respectively the following

$$n_s = 1 - 6\epsilon + 2\eta\,, \qquad r = 16\epsilon\,,$$

while the amplitude of the primordial power spectrum is

$$\Delta^2_{\mathcal{R}} = \frac{\kappa^2 H^2}{8\pi^2 \epsilon}\,,$$

We obtain that $n_s \approx 1.01$ and $r \approx 0.10$ that seem in agree with the value of spectral index estimated from Planck+ WP data that is n_s = 0.96030±0073 (68% C.L.) and r < 0.11 (95% CL) (Planck collaboration, Astron. Astrophys. 571 (2014) A22)

This results are also consistent with the value measured by the BICEP2 collaboration (Ade et al. arXiv:1403.3985 (2014)).

Power law inflation

Equation which governs the evolution of the matter fluctuations in the linear regime

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m = 0,$$

$$G_{\text{eff}} = \frac{G_N}{F_R(R,\mathcal{G})},$$

We use the equation of energy with matter density contribution as

$$4\pi G\rho_{(m)} = \frac{3H^2}{2} - 4\pi G\rho_{(\mathcal{G}B)} \quad \text{with} \quad \rho_{(\mathcal{G}B)} = \frac{RF_R - F(R,\mathcal{G}) - 6H\dot{F}_R + \mathcal{G}F_{\mathcal{G}} - 24H^3\dot{F}_{\mathcal{G}}}{16\pi G_N}$$

we obtain the equation $\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{RF_R - F(R,\mathcal{G}) - 6H\dot{F}_R + \mathcal{G}F_{\mathcal{G}} - 24H^3\dot{F}_{\mathcal{G}}}{4F_R}$

$\delta_m = 0.$

Remembering that $a(t) = a_0 t^s = a_0 t^{2n-1}$ and consequently $H = 2n-1/t$

$$\ddot{\delta}_m + \frac{2n-1}{t}\dot{\delta}_m + \frac{3(6n^2-6n-1)}{2t^2}\delta_m = 0.$$

Power law inflation

The above equation is an Euler equation of which the general
solution is

$$\delta_m(t) = t^{\frac{1}{2}(-\sqrt{3-8n^2}-4n+3)}\left(c_2 t^{\sqrt{3-8n^2}} + c_1\right)$$

Since a(z)= $(1 + z)^{-1}$ we have that

$$H = H_0 a^{-\frac{1}{2n-1}} = H_0 \left(\frac{1}{1+z}\right)^{\frac{1}{2n-1}}.$$

The deceleration parameter q is

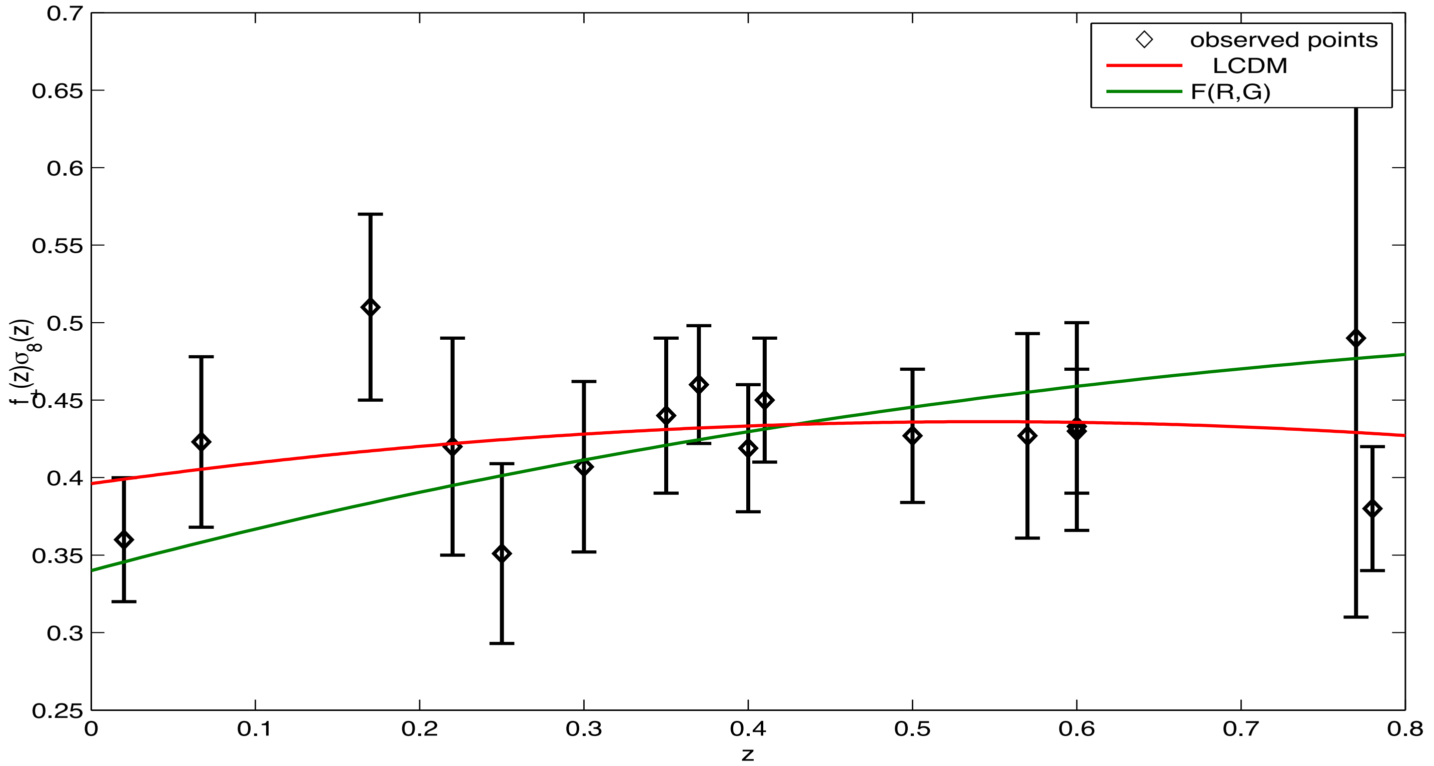
$$q = -1 - \frac{\mathrm{d}\ln H}{\mathrm{d}\ln a} = -1 + \frac{1}{2n-1}$$

Clearly, for n = 1, the solution is an Einstein–de Sitter model as it has to be.

On the other hand, the accelerated expansion of the universe (q < 0) is recovered for n > 1, but, in this case, the universe accelerates forever without the possibility of structure formation

S. Basilakos, S. Capozziello, M. De Laurentis, A. Paliathanasis, and M. Tsamparlis, Phys. Rev. D 88, 103526 (2013)

Power law inflation



Conclusions and remarks

- the Gauss Bonnet invariant has a main role in renormalization of fourth-order gravity
- F(R,G) cosmology suitably extends the Starobinsky cosmology
- F(R,G) gravity gives rise to a double inflation related to the effective fields R and G
- These results could be important for large scale and very large scale structures