# Anisotropic Power Spectrum from Rotational Symmetry Breaking Excited Initial States

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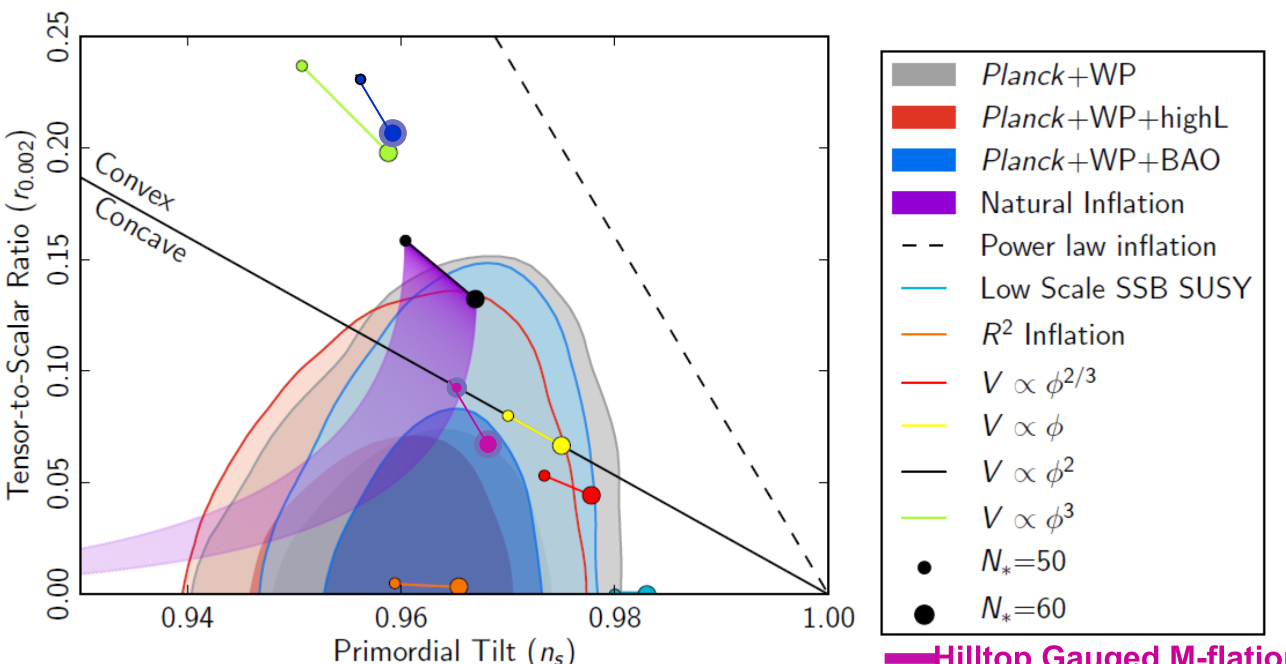
## ■Introduction & Outline

**The increasingly precise CMB measurements by Planck mission in combination with other cosmological date have ushered us into a precision early Universe cosmology era:**

$$n_S \equiv \frac{d\log P_S}{d\log k} + 1 = 0.968 \pm 0.006$$

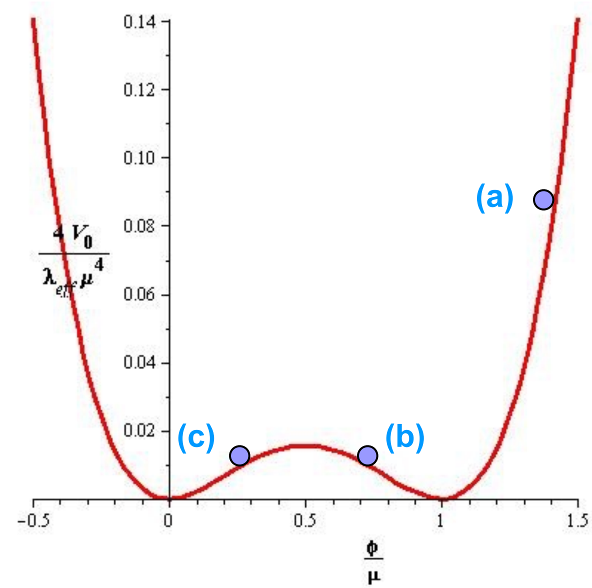
**Planck 2015**

$$r < 0.11$$




**Hilltop Gauged M-flation**

**SB Gauged M-flation**




**Ashoorioon & Sheikh-Jabbari (2009, '11,'13)**

## Introduction & Outline

- Inflation is based on QFT and GR
- Some symmetries of low energy physics: Rotational Invariance, Lorenz Invariance
- $1 \text{ MeV} \leq M_{\text{Inf.}} \leq 10^{16}\text{GeV}$
- What if some of these symmetries are broken before or during Inflation?
- In this work we assume that EFT is valid up to *M* where $M_{\text{Inf}} < M$
- The effect of high energy theories above *M* is to excite the vacuum to a rotationally breaking excited initial state.
- Also we shall consider the case of a preferred direction in momentum space, $\hat{n}$

$$\mathcal{P}_S = \mathcal{P}_{\text{iso}} \left[1 + M(\hat{k})\right] \longrightarrow \Delta T(\hat{k}) = \Delta T_{\text{iso}}(\hat{k}) \left[1 + M(\hat{k})\right] \; .$$

$$M(\hat{k}) = A\, \hat{k} \cdot \hat{n} + B\, (\hat{k} \cdot \hat{n})^2 + C\, (\hat{k} \cdot \hat{n})^3 + \ldots \; ,$$

**A, C, … (odd multipoles) have to be pure imaginary numbers**

$A$ → **dipole**, $B$ → quadrupole, $C$ → Octupole

## Introduction & Outline

- Planck 2013, bounds the quadrupolar term using the bounds on NG in the context of anisotropic inflationary models

$$-0.05 < B < 0.05 \ (95\% \ \text{C.L.}).$$

model-dependent

- Kim & Komatsu (2013), doing data analysis on the Planck 2013 data

$$-0.03 < B < 0.033 \ (95\% \ \text{C.L.})$$

- We then find the signature of such excited initial states in the bispectrum

- We also find an analytic bound on parameter *B*

$$|B| < 0.06$$

which is comparable with the above results.

## ROTATIONAL SYMMETRY BREAKING EXCITED INITIAL STATES

- The equation for gauge-invariant scalar perturbations

$$u''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right) u_{\vec{k}} = 0 \qquad z \equiv \frac{a\phi'}{\mathcal{H}}, \quad \mathcal{H} \equiv \frac{a'}{a}$$

$$u = -z\left(\frac{a'}{a}\frac{\delta\phi}{\phi'} + \Psi\right)$$

- In a quasi-deSitter background

$$a(\tau) \simeq -\frac{1}{H\tau}$$

the most generic solution to the E.O.M. in the leading order in slow-roll parameters

$$u_{\vec{k}}(\tau) \simeq \frac{\sqrt{\pi|\tau|}}{2}\left[\alpha^*_{\vec{k}} H^{(1)}_{3/2}(k|\tau|) + \beta^*_{\vec{k}} H^{(2)}_{3/2}(k|\tau|)\right] ,$$

where the Bogoliubov coefficients satisfy the Wronskian condition

$$|\alpha^*_{\vec{k}}|^2 - |\beta^*_{\vec{k}}|^2 = 1$$

## Backreaction and bounds on $|\beta_{\vec{k}}|$

- Any excited state contains massless quanta whose positive pressure can derail slow-roll Inflation

  one can see derailing the slow-roll inflation can be avoided

$$\delta\rho_{\text{non-BD}} \ll \epsilon\,\rho_0$$

$$\delta p'_{\text{non-BD}} \ll \mathcal{H}\,\eta\,\epsilon\,\rho_0$$

The second equation, which is the stronger one, can be written as

$$\int_H^\infty \frac{\mathrm{d}^3k}{(2\pi)^3}\, k|\beta^*_{\vec{k}}|^2 \ll \epsilon\,\eta\,H^2 M_{\text{pl}}^2$$

- As a specific example, let us consider the crude model in which the modes get excited when $k/a(\tau) = M$

$$\beta_{\vec{k}} = \beta_0(\hat{k})$$

one obtains the following bound on $\beta_0$

$$|\beta_0(\hat{k})| \lesssim \sqrt{\epsilon\,\eta}\,\frac{H M_{\text{Pl}}}{M^2} \sim \epsilon\,\frac{H M_{\text{Pl}}}{M^2}$$

# Effect of an Excited State in the Power Spectra

- **Scalar power spectrum**

Ashoorioon, Dimopoulos, Sheikh-Jabbari & Shiu (2013)

$$\mathcal{P}_S = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2_{k/\mathcal{H}\to 0} \qquad \mathcal{P}_S = \mathcal{P}_{\rm BD}\,\gamma_S$$

$$\mathcal{P}_{\rm BD} = \frac{1}{8\pi^2\epsilon}\left(\frac{H}{M_{\rm pl}}\right)^2 , \qquad \gamma_S = |\alpha^*_{\vec{k}} - \beta^*_{\vec{k}}|^2_{k=\mathcal{H}} \ .$$

- **Parameterization of the Parameter Space**

$$|\alpha^*_{\vec{k}}|^2 - |\beta^*_{\vec{k}}|^2 = 1 \Longrightarrow \alpha^S_{\vec{k}} = e^{i\varphi_S}\cosh\chi_S \ , \qquad \beta^S_{\vec{k}} = e^{-i\varphi_S}\sinh\chi_S \ ,$$

Let us focus on $V(\phi) = \frac{1}{2}m^2\phi^2$

Using the Planck normalization for the amplitude of density perturbations:

$$\frac{H}{M_{pl}} \simeq \frac{1}{\sqrt{\gamma_S}} 3.78\times 10^{-5}$$

that with the help of backreation condition, $\beta_0^S \le \frac{\epsilon H M_{pl}}{M^2}$, yields

$$\frac{M^2}{H^2} \lesssim 220\,\frac{\sqrt{\gamma_S}}{\sinh\chi_S}$$

## Effect of an Excited State in the Power Spectra

- **Quasi-BD region, $\chi_S \ll 1$ and general $\varphi_S$:**
  - $M$ can be arbitrary large
  - $H$ is very close to its Bunch-Davies value

- **Typical or large values of $\chi_S$, $\chi_S \gtrsim 1$ :**

  - $\sqrt{\gamma_S} \simeq e^{\chi_S} \sin(\varphi_S)$
  - $\sinh \chi_S \simeq \frac{e^{\chi_S}}{2}$
  - generic values of $\varphi_S$

  $\Longrightarrow$

  - $M \lesssim 21H$
  - $H \lesssim H_{BD}$

- Desirable value of $M \simeq 21\, H$ is obtained if $\varphi_S \simeq \frac{\pi}{2}$.

## ■ Rotational Breaking Excited Initial States:

- **Parameterization and Power Spectrum**

$$\beta_0(\hat{k}) = \sinh\left(\chi_s + \varepsilon_2\, c_{\hat{k}}^2\right) e^{-i\left(\varphi_S + \delta_2\, c_{\hat{k}}^2\right)}$$

$$\hat{k}\cdot\hat{n} \equiv \cos\psi_{\vec{k}} \equiv c_{\hat{k}}$$

$$\alpha_0(\hat{k}) = \cosh\left(\chi_s + \varepsilon_2\, c_{\hat{k}}^2\right) e^{i\left(\varphi_S + \delta_2\, c_{\hat{k}}^2\right)}$$

$$\rho_{\text{non-BD}} = \frac{1}{a^3}\int_0^\infty \frac{\mathrm{d}^3k}{(2\pi)^3}\sqrt{m^2 + \left(\frac{k}{a}\right)^2}\,|\beta_{\vec{k}}|^2 = \frac{M^4}{4\pi^2}\sinh^2(\chi_S)\, f_{\frac{m}{M}} f_{\varepsilon_2}\,.$$

$$f_{\frac{m}{M}} \simeq 1 + \frac{m^2}{12M^2}$$

$$m \ll M$$

$$\lim_{\chi_S\to\infty} f_{\varepsilon_2} = \frac{1}{2}\sqrt{\frac{\pi}{2\varepsilon_2}}\,\mathrm{Erfi}\left(\sqrt{2\varepsilon_2}\right)$$

$$\left(\frac{M}{H}\right)^4 \lesssim \frac{\gamma_S}{2\sinh^2\chi_S\, f_{\frac{m}{M}} f_{\varepsilon_2}}\left[\frac{\eta}{\mathcal{P}_S}\right]_0$$

$$\lim_{\chi_S\to\infty}\frac{\gamma_S}{2\sinh^2\chi_S} = 1 - \cos 2\varphi_S\,,$$

The upper bound is again maximal when $\varphi_S = \frac{\pi}{2}$

## Rotational Breaking Excited Initial States:

$$\left(\frac{M}{M_{\rm Pl}}\right)^4 \lesssim \frac{32\pi^2}{\gamma_S \sinh^2\chi_S f_{\frac{m}{M}} f_{\varepsilon_2}} [\epsilon\eta\mathcal{P}_S]_0 \qquad \lim_{\chi_S\to\infty} \frac{1}{2\gamma_S\sinh^2\chi_S} \to 0$$

This bound restricts us to consider ultra-low scale inflation with increasing $\chi_S$

**Maximally Occupied Vacuum $\chi_S \gg 1$**

- In this limit the factor *B* could be read expanding the $\gamma_S$ factor in $\varepsilon_2$ and $\delta_2$

$$B = \varepsilon_2\left(2 - 4e^{-2\chi_S} + 4e^{-4\chi_S}\cot^2\varphi_S\right) + \delta_2\left(-2e^{-4\chi_S}\cot\varphi_S\csc^2\varphi_S + 2\cot\varphi_S\right)$$

  - In the $\chi_S \gg 1$ where $\varphi_S \simeq \frac{\pi}{2}$

$$A = 0 \qquad B \simeq 2\varepsilon_2 \qquad C = 0$$

- Now from the observation constraint on *B,* the following constraint is obtained on $\varepsilon_2$

$$-0.015 < \varepsilon_2 < 0.0165 \ (95\% \text{ C.L.})$$

$\delta_2$ remains indefinite in this regime from the constrains on the quadrupole moment.

## ■ Rotational Breaking Excited Initial States:

- **Bispectrum:**

One can compute the bispectrum using the in-in formalism:

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = -i\,(2\pi)^3\delta^3\left(\sum_{i=1}^{3}\vec{k}_i\right)\left(\frac{\dot{\phi}}{H}\right)^4\frac{H}{M_P^2}$$
$$\times\int_{\tau_0}^{0}\frac{\mathrm{d}\tau}{k_3^2}\left[a(\tau)\,\partial_\tau G^{>}_{\vec{k}_1}(0,\tau)\right]$$
$$\times\left[a(\tau)\,\partial_\tau G^{>}_{\vec{k}_2}(0,\tau)\right]\left[a(\tau)\,\partial_\tau G^{>}_{\vec{k}_3}(0,\tau)\right]$$
$$+\text{permutations}+\text{c.c.}\,, \qquad (59)$$

where the Whightman function is

$$G^{>}_{\vec{k}}(\tau,\tau') \equiv \frac{H^2}{\dot{\phi}^2}\frac{u_k(\tau)}{a(\tau)}\frac{u_k^*(\tau')}{a(\tau')}\,,$$

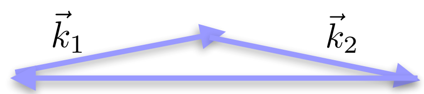
# Rotational Breaking Excited Initial States:

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = (2\pi)^3\delta^3\left(\sum_{i=1}^{3}\vec{k}_i\right)\frac{2H^6\sum_{i>j}k_i^2k_j^2}{\dot{\phi}^2M_P^2\prod_{i=1}^{3}(2k_i^3)} \times\left[\mathscr{A}\frac{1-\cos(k_t\eta_0)}{k_t}+\mathscr{B}\frac{\sin(k_t\eta_0)}{k_t} +\sum_{j=1}^{3}\mathscr{C}_j\frac{1-\cos(\tilde{k}_j\eta_0)}{\tilde{k}_j}+\sum_{j=1}^{3}\mathscr{D}_j\frac{\sin(\tilde{k}_j\eta_0)}{\tilde{k}_j}\right] \quad (61)$$

$$\begin{aligned}
\mathscr{A} &= \prod(\alpha_{\vec{k}_i}-\beta_{\vec{k}_i})\left(\prod\alpha^*_{\vec{k}_i}+\prod\beta^*_{\vec{k}_i}\right)+\text{c.c.}\\
\mathscr{B} &= i\prod(\alpha_{\vec{k}_i}-\beta_{\vec{k}_i})\left(\prod\beta^*_{\vec{k}_i}-\prod\alpha^*_{\vec{k}_i}\right)+\text{c.c.}\\
\mathscr{C}_j &= \prod(\beta_{\vec{k}_i}-\alpha_{\vec{k}_i})\left(\frac{\beta^*_{\vec{k}_j}}{\alpha^*_{\vec{k}_j}}\prod\alpha^*_{\vec{k}_i}+\frac{\alpha^*_{\vec{k}_j}}{\beta^*_{\vec{k}_j}}\prod\beta^*_{\vec{k}_i}\right)+\text{c.c.}\\
\mathscr{D}_j &= i\prod(\alpha_{\vec{k}_i}-\beta_{\vec{k}_i})\left(\frac{\beta^*_{\vec{k}_j}}{\alpha^*_{\vec{k}_j}}\prod\alpha^*_{\vec{k}_i}-\frac{\alpha^*_{\vec{k}_j}}{\beta^*_{\vec{k}_j}}\prod\beta^*_{\vec{k}_i}\right)+\text{c.c.}
\end{aligned} \quad (62)$$

There are two types of enhancement in presence of excited initial states:

- Flattened configurations, $k_1 + k_2 \simeq k_3$




X. Chen, et. al (2005)

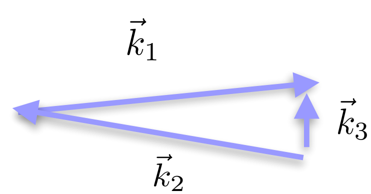
This enhancement is lost after projection on the 2D CMB surface!

Holman & Tolley (2007)

- Local configuration, $k_1 \simeq k_2 \gg k_3$

Agullo & Parker (2010)

$$f_{\rm NL} \sim \epsilon\frac{k_S}{k_L}$$



## ■ Rotational Breaking Excited Initial States:

using the definition $f_{\rm NL} \equiv -\frac{5}{6}\frac{\delta\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle}{\sum_{i>j}\langle\zeta_{\vec{k}_i}\zeta_{\vec{k}_i}\rangle\langle\zeta_{\vec{k}_j}\zeta_{\vec{k}_j}\rangle}$

$$f_{\rm NL} = -\frac{20}{3}\,\epsilon\,\frac{k_1}{k_3}\,\frac{\mathfrak{C}}{\gamma_S(\vec{k}_3)[\gamma_S(\vec{k}_1)+\gamma_S(\vec{k}_2)]}$$

$$\mathfrak{C} = \Re\left\{\prod_{i=1}^{3}(\alpha_{\vec{k}_i}-\beta_{\vec{k}_i})\left[\prod_{i=1}^{3}\alpha^*_{\vec{k}_i}\left(\frac{\beta^*_{\vec{k}_1}}{\alpha^*_{\vec{k}_1}}+\frac{\beta^*_{\vec{k}_2}}{\alpha^*_{\vec{k}_2}}\right) + \prod_{i=1}^{3}\beta^*_{\vec{k}_i}\left(\frac{\alpha^*_{\vec{k}_1}}{\beta^*_{\vec{k}_1}}+\frac{\alpha^*_{\vec{k}_2}}{\beta^*_{\vec{k}_2}}\right)\right]\right\}$$
$$= \Re\left[\prod_{i=1}^{3}(\alpha_{\vec{k}_i}-\beta_{\vec{k}_i})(\alpha^*_{\vec{k}_3}+\beta^*_{\vec{k}_3})(\alpha^*_{\vec{k}_2}\beta^*_{\vec{k}_1}+\alpha^*_{\vec{k}_1}\beta^*_{\vec{k}_2})\right]$$

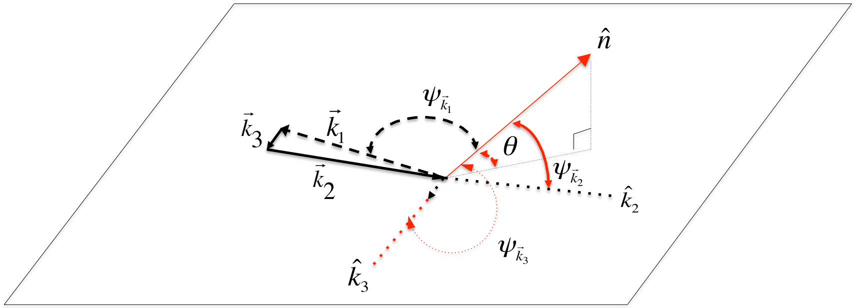
Expanding in terms of $\varepsilon_2$ and $\delta_2$ and in the $\chi_S \gg 1, \varphi_S \simeq \frac{\pi}{2}$

$$f_{\rm NL} \simeq f^0_{\rm NL} + f^{\varepsilon_2}_{\rm NL}\varepsilon_2 + f^{\delta_2}_{\rm NL}\delta_2$$

$$f^0_{\rm NL} \simeq \frac{5\epsilon}{3}\frac{k_1}{k_3}$$
$$f^{\varepsilon_2}_{\rm NL} \simeq \frac{5\epsilon}{3}\frac{k_1}{k_3}\left[c^2_{\vec{k}_1}+c^2_{\vec{k}_2}-2\,c^2_{\vec{k}_3}\right]$$
$$f^{\delta_2}_{\rm NL} \simeq 0\ .$$

$$c_{\vec{k}_i} \equiv \cos\psi_{\vec{k}_i} = \hat{k}_i\cdot\hat{n}$$

## Rotational Breaking Excited Initial States:




$$k_1 \simeq k_2 \gg k_3$$

$$\psi_{\vec{k}_2} \approx \psi_{\vec{k}_1} + \pi$$

$$\theta \lesssim \psi_{\vec{k}_i} \lesssim \pi - \theta$$

$$\cos^2 \psi_{\vec{k}_1} + \cos^2 \psi_{\vec{k}_3} \simeq \cos^2 \theta$$

$$f_{\rm NL} = \frac{5}{3}\,\epsilon\,\frac{k_1}{k_3}\left[1 + \varepsilon_2\left(4\cos^2\psi_{\vec{k}_1} - 2\cos^2\theta\right)\right]$$

For given $\theta$ the maximum enhancement occurs when $\psi_{\vec{k}_1} = \theta$

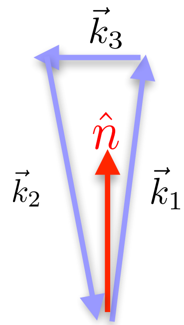
The minimum occurs when $\psi_{\vec{k}_1} = \dfrac{\pi}{2}$

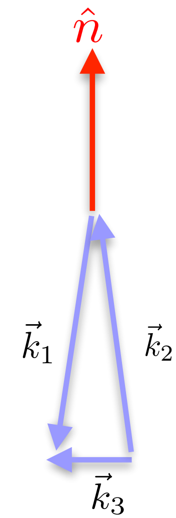
This variation enhances for the local configuration that is coplanar with $\hat{n}$

## Rotational Breaking Excited Initial States:

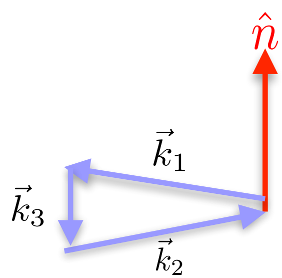
$\vec{k}_1$ corresponding to shortest scales probed by Planck and $\vec{k}_3$ corresponding to largest scale at which the cosmic variance is negligible, $l \simeq 10$. For $\epsilon \simeq 0.01$ and $\varepsilon_2 \simeq 0.0165$

$$f_{\rm NL}^{\rm max} \simeq 4.3$$




$$f_{\rm NL}^{\rm min} \simeq 4.03$$




$$\Delta f_{\rm NL} \simeq 0.27$$

## ■ Rotational Breaking Excited Initial States:

✱ Purely anisotropic Initial Condition, $\chi_S \ll 1$

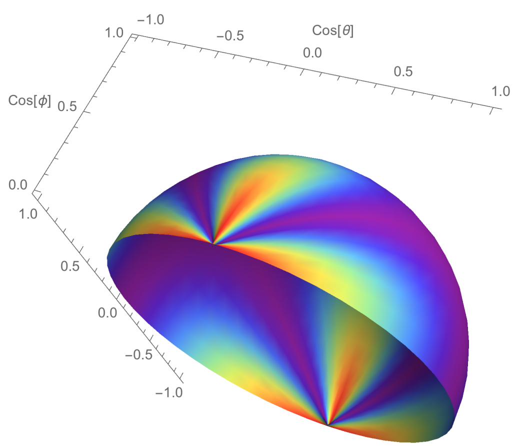
In this case: $A = 0$ $B = -2\cos\varphi_S \varepsilon_2$ $C = 0$

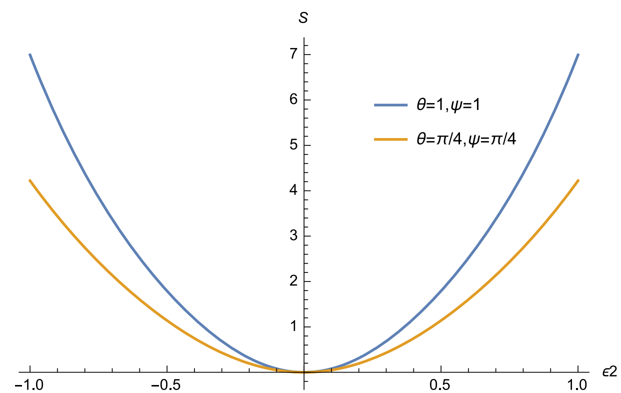
- One intriguing case is when $\varphi_S = \frac{\pi}{4}$ ⟹ $B = 0$ no bound from quadrupole term on $\varepsilon_2$

- Still to be able to to trust the expansion in $\varepsilon_2$ and $\delta_2$ we assume $0 \ll \varepsilon_2 \lesssim 1$

We also set $\delta_2 = 0$ .

$$f_{NL}^{(0)} = \frac{5\epsilon k_1}{3k_3} \mathrm{S}(\theta, c^2_{\hat{k}_1})$$




bipolar bispectrum that can reach $f_{\rm NL} \simeq 20 - 30$

## Conclusion

- Effect of new physics or pre-inflation can be encoded as the excited states at the new physics hypersurface, $\frac{k}{a(\tau)} \simeq M$
- If the new physics or pre-inflation evolution break the rotational invariance by picking up a preferred direction in such an excited states, the resulted power spectrum can turn out to be anisotropic.
- We constrained the form of such rotational breaking excited initial state using the CMB observation.
- We also found the signature of the model in the bispectrum.
- In general the form of the bispectrum will depend on the direction of the momenta.
- The local non-gaussianity is enhanced in general.

## Conclusion

- In the maximally excited initial state, the maximum of local NG occurs when the short wavelength modes are collinear or anti collinear with the preferred direction. The minimum occurs when these modes are perpendicular to the preferred direction.

- In the purely anisotropic initial condition for the specific configuration where $\varphi_S = \pi/4$ the bispectrum becomes bipolar where $f_{\rm NL} \simeq 20 - 30$. This extremely anisotropic feature of the non-Gaussianity will be the signature of the model in this region of parameter space.

*Thank you!*

## Bound on the quadrupole moment, $B$:

So we have an anisotropic power spectrum

$$\mathcal{P}(k) = \mathcal{P}_{\rm iso}(k)\left(1 + B(\hat{k}\cdot\hat{n})^2\right)$$

As mentioned the strongest constraint was provided by Kim & Komatsu (2013)

$$-0.03 < B < 0.033 \ (95\% \ \text{C.L.})$$

ing the data analysis techniques on the Planck 2013 data. However there is a
y to obtain a bound on *B* analytically:

$$\frac{\Delta T}{T}(\hat{e}) = \int \mathrm{d}\vec{k} \sum_l \left(\frac{2l+1}{4\pi}\right)(-i)^l \, P_l(\hat{k}\cdot\hat{e})\, \mathcal{R}(\vec{k})\, \Theta_l(k)$$

$$a_{lm} = \int \mathrm{d}\Omega_{\hat{e}} \, [Y_l^m(\hat{e})]^* \frac{\Delta T}{T}(\hat{e}) \ .$$

$$\langle a_{lm}\, a^*_{l'm'}\rangle = \langle a_{lm}\, a^*_{l'm'}\rangle_0 + \Delta(lm;l'm')$$

$$\langle a_{lm}\, a^*_{l'm'}\rangle_0 = \delta_{ll'}\,\delta_{mm'} \int_0^\infty \frac{\mathrm{d}k}{k}\, \mathcal{P}_{\rm iso}(k)\, \Theta_l^2(k) \ .$$

$$n_+ = -\left(\frac{n_x - i\,n_y}{\sqrt{2}}\right) \ , \ n_- = \left(\frac{n_x + i\,n_y}{\sqrt{2}}\right) \ , \ n_0 = n_z$$

$$\Delta(lm;l'm') = (-i)^{l-l'} \times \int_0^\infty \frac{\mathrm{d}k}{k}\, \mathcal{P}_{\rm iso}(k) B\, \xi^{(2)}_{lm;l'm'} \Theta_l(k)\, \Theta_{l'}(k) \ ,$$

$$\xi^{(2)}_{lm;l'm'} = n_+^2\, \xi^{++(2)}_{lm;l'm'} + n_-^2\, \xi^{--(2)}_{lm;l'm'} + 2\,n_+\,n_-\, \xi^{+-(2)}_{lm;l'm'} + 2\,n_+\,n_0\, \xi^{+0(2)}_{lm;l'm'} + 2\,n_-\,n_0\, \xi^{-0(2)}_{lm;l'm'} + n_0^2\, \xi^{00(2)}_{lm;l'm'} \ .$$

## ■ Bound on the quadrupole moment, $B$:

$$\xi^{--}_{lm;l'm'} = -\delta_{m',m+2} \times \Bigg[\delta_{l',l}\frac{\sqrt{(l^2-(m+1)^2)(l+m+2)(l-m)}}{(2l+3)(2l-1)} -\frac{1}{2}\delta_{l',l+2}\sqrt{\frac{(l+m+1)(l+m+2)(l+m+3)(l+m+4)}{(2l+1)(2l+3)^2(2l+5)}} -\frac{1}{2}\delta_{l',l-2}\sqrt{\frac{(l-m)(l-m-1)(l-m-2)(l-m-3)}{(2l+1)(2l-1)^2(2l-3)}}\Bigg],$$

$$\xi^{+-}_{lm;l'm'} = \frac{1}{2}\delta_{m',m}\Bigg[-2\,\delta_{l',l}\frac{(-1+l+l^2+m^2)}{(2l-1)(2l+3)} + \delta_{l',l+2}\sqrt{\frac{((l+1)^2-m^2)((l+2)^2-m^2)}{(2l+1)(2l+3)^2(2l+5)}} + \delta_{l',l-2}\sqrt{\frac{(l^2-m^2)((l-1)^2-m^2)}{(2l-3)(2l-1)^2(2l+1)}}\Bigg],$$

$$\xi^{-0}_{lm;l'm'} = -\frac{1}{\sqrt{2}}\delta_{m',m+1}\Bigg[\delta_{l',l}\frac{(2m+1)\sqrt{(l+m+1)(l-m)}}{(2l-1)(2l+3)} +\delta_{l',l+2}\sqrt{\frac{((l+1)^2-m^2)(l+m+2)(l+m+3)}{(2l+1)(2l+3)^2(2l+5)}} -\delta_{l',l-2}\sqrt{\frac{(l^2-m^2)(l-m-1)(l-m-2)}{(2l-3)(2l-1)^2(2l+1)}}\Bigg],$$

$$\xi^{++}_{lm;l'm'} = \xi^{--}_{l'm';lm},$$

$$\xi^{+0}_{lm;l'm'} = -\xi^{-0}_{l'm';lm},$$

The effect of quadrupolar term in the primordial spectrum is correlating $a_{lm}$ with $a_{l\pm0\{\text{or }1\},m\pm0\{\text{or }1\}}$ and $a_{l\pm0\{\text{or }1\},m\pm0\{\text{or }1\}}$

## Bound on the quadrupole moment, $B$:

Assuming $B(k) = B_*$

Ackerman, Carroll & Wise (2007)

$$\frac{\Delta(lm;lm)}{\langle a_{lm}\, a^*_{lm}\rangle_0} = \frac{B_*}{2}\left[\sin^2\theta_* + (3\cos^2\theta_* - 1)\,\frac{2l^2+2l-2m^2-1}{(2l-1)(2l+3)}\right]$$

$a_{lm}$ are independent random variables

$$\langle a_{lm}\, a^*_{lm}\rangle_0 = C^0_l\, \delta_{ll'}\, \delta_{mm'}$$

$$\frac{\Delta C_l}{C_l} \simeq \frac{\Delta C_l}{C^0_l} = \frac{B_*}{2}\left[\sin^2\theta_* + (3\,\cos^2\theta_* - 1)\,\frac{(2l+1)(2l-3)}{3(2l-1)(2l+3)}\right]$$

We have not seen any deviation from cosmic variance at large *l*'s

$$\frac{\Delta C_l}{C_l} \leq \left.\frac{\Delta C_l}{C_l}\right|_{\mathrm{s.v.}} = \sqrt{\frac{2}{2l+1}}$$

Planck has probed up to $l \simeq 2500$. At large *l*'s we also the bound on $B_*$ does not depend on $\theta_*$

$$|B_*| \lesssim 0.06$$