Kerr-Schild Way to Quantum Gravity

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Based on:

A.B., Gravitating Lepton Bag Model, JETP, v.148 (8), 228 (2015), ibid. v.148(11), 937 (2015), [arXiv:1505.03439].

A.B., Wonderful Consequences of the Kerr Theorem, Grav. and Cosmol, 11, 301 (2005), [arXiv:hep-th/0506006].

A.B., Grav. and Cosmol., 21 (1), 28 (2015) [arXiv:1404.5947].

The Kerr-Schild (KS) approach to QG is based on the KS form of metric

$$g_{\mu\nu} = \eta^{\mu\nu} + 2Hk^\mu k^\nu \tag{1}$$

and the Kerr Theorem, which determines the null vector field $k^\mu, (k^\mu k_\mu = 0)$– *the Kerr Congruence*, the structure of function H and basic parameters of the exact KS solutions of the Einstein-Maxwell field equations.

KS Gravity has some peculiarities indicating his profound differences from normal gravity, and simultaneously his inner connection with the quantum theory. Among them are:

- two-sheeted space-time,

- beam-like form of elementary excitations,

- twistorial structure forms the light-like network solutions of the Einstein-Maxwell equations,

- gyromagnetic ratio (of the Kerr-Newman (KN) solution) is $g = 2$ as that of the Dirac electron,

- Wilson loop leading to quantization of the spin and mass,

- stringy structures,

- consistent embedding of the Dirac equation,

- KS gravity *"knows"* the Compton length and the fine structure constant. How did it know that?

Contrary to effective gravitation, the KS gravity is considered as *theory of space*, which should provide effective work of Qyantum theory.

TWOSHEETED space – the in- and out-metrics $g^{\pm}_{\mu\nu} = \eta_{\mu\nu} + 2Hk^{\pm}_{\mu}k^{\pm}_{\nu}$ **on the same Minkowski background** $\eta^{\mu\nu}$.

PROBLEM OF SOURCE OF THE KERR SOLUTION, since 1965.

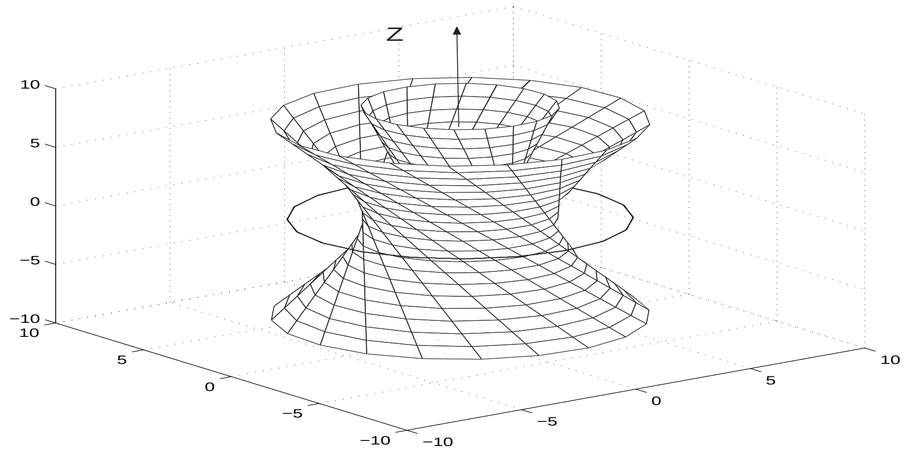


Figure 1: **Ultra-rotating Kerr solution: singular ring and twistor bundle of Kerr congruence.**

Black holes as elementary particles: Salam-Strathdee(1976); G't Hooft(1990); V.Frolov, M.Markov, V.Mukhanov(1990); A.Sen(1995); Holzhey-Wilczek(1992).

Kerr-Newman solution as a model of electron.

as consequence of $g = 2$, **(B.Carter, 1968.) the gravitational and EM fields of the KN solution correspond to electron. Extreme large spin/mass ratio:** $a = J/m >> m$. $\Rightarrow$ ***Ultra-extreme* solution without horizons! Two-sheeted space and topological defect prevent the effective work of Quantum Theory.**

Kerr-Newman solution as a model of electron:

W.Israel, Source of the Kerr metric *Phys. Rev.* **D 2** 641 (1970).
A.Burinskii, Microgeons with spins *Sov. Phys. JETP* **39** 193 (1974).
V.Hamity, An interior of the Kerr metric, *Phys. Lett.* A **56**, 77, (1976).
C.A. López, An extended model of electron in general relativity *Phys.Rev.***D30** 313 (1984).
A.Burinskii, Spinning Particle as a Non-trivial Rotating Super Black Hole with Broken N=2 Supersymmetry, *Class.Quant.Grav.***16**, 3497 (1999).
A.Burinskii, E.Elizalde, S.R.Hildebrandt, G.Magli, Regular Sources of the Kerr-Schild Class for Rotating and Nonrotating Black Hole Solutions, *Phys.Rev.***D65**, 064039 (2002).
A.Burinskii, Kerr-Newman electron as spinning soliton,*IJMP* **A29** 1450133 (2014).

Relations with (super)string theory:

D.D.Ivanenko and A.Ya. Burinskii, **Gravitational strings in the models of elementary particles,** *Izv. Vuz. Fiz.*, **5**, 135 (1975).
A.Burinskii, Some properties of the Kerr solution to **low-energy string theory,** *Phys.Rev.* **D 52** 5826 (1995).

Bag model beyond SM:

A.Burinskii, Gravitating Lepton Bag Model, *JETP,* v.148 (8), 228 (2015).
A.Burinskii, Stability of the Lepton Bag Model Based on..., *JETP,* v.148(11), 937 (2015).
A.Burinskii, Source of the Kerr-Newman Solution as a Supersymmetric Domain-Wall Bubble: 50 years of the problem, *Phys.Letters,* B 754 (2016) 99

Separation of the quantum zone from external gravity.

Generalization of the classical electron: ROTATING BUBBLE (López 1984)

SHAPE OF the bubble is uniquely determined by the KN metric

$$g_{\mu\nu}^{(KN)} = \eta_{\mu\nu} + 2H_{(KN)} k_\mu k_\nu, \qquad (2)$$

where $H_{(KN)} = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}$.

"ZERO GRAVITY SURFACE": $H_{(KN)} = 0 \quad \Rightarrow \quad r = R = e^2/2m$, **as boundary of the bubble.**

r **is oblate spheroidal coordinate.**

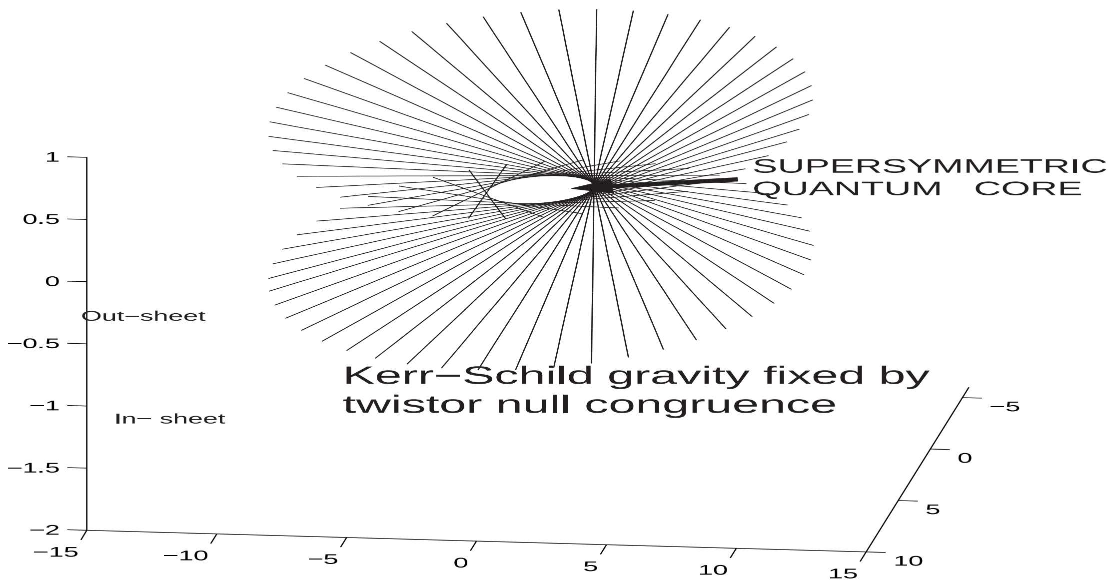
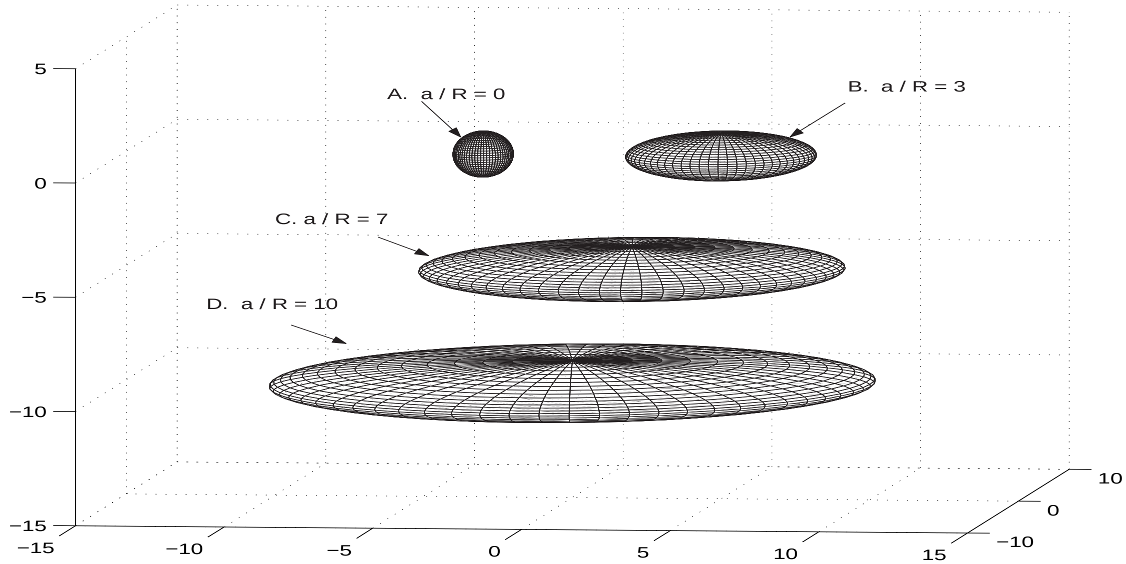


Figure 2: **Flat core is necessary to ensure consistency with quantum theory**.

Degree of oblateness of the bubble (thickness/radius = R/a) depends on the Kerr spin parameter $a = J/m$. For an electron $R/a = \alpha = 137^{-1}$ it is the fine structure constant.



The bubble-source is stretched by rotation to the *disk of the Compton radius* corresponding to *a dressed electron* ($R/a \sim 137^{-1}$).

DOMAIN WALL separates the flat QUANTUM zone from external KN GRAVITY. Conflict with gravity is removed by using of supersymmetric scheme of phase transition [A.B. JETP 148(2),228 (2015)].

SUPERSYMMETRIC scheme of phase transition.

Triplet of the chiral fields $\Phi^{(i)} = \{H, Z, \Sigma\}$, where H is the Higgs field.

Lagrangian $\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{3} F^{(i)}_{\mu\nu}F^{(i)\mu\nu} - \frac{1}{2}\sum_{i=1}^{3}(\mathcal{D}^{(i)}_{\mu}\Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^{*} - V$, covariant derivatives $\mathcal{D}^{(i)}_{\mu} = \nabla_{\mu} + ieA^{(i)}_{\mu}$.

Superpotential (suggested by J. Morris, 1996)

$$W = \Phi^{(2)}(\Phi^{(1)}\bar{\Phi}^{(1)} - \eta^2) + (\Phi^{(2)} + \mu)\Phi^{(3)}\bar{\Phi}^{(3)}, \tag{3}$$

determines the potential

$$V(r) = \sum_{i} |\partial_i W|^2, \tag{4}$$

where $\mathcal{H} \equiv \Phi^{(1)}$ is taken as Higgs field.

Vacuum states $V_{(vac)} = 0$ are determined by the conditions $\partial_i W = 0$. The model yields two vacuum states:

(I) the supersymmetric false-vacuum state inside: $|\mathcal{H}| = \eta;\ Z = -\mu;\ \Sigma = 0$,

(II) the vacuum state outside: $|\mathcal{H}| = 0;\ Z = 0;\ \Sigma = \eta$.

Higgs field $\mathcal{H}$ forms inside the bag a supersymmetric false-vacuum state.
Einstein-Maxwell eqs. are trivially satisfied inside and outside the bag.

Basic equations for interaction of the complex Higgs field $\mathcal{H}(x) = |\mathcal{H}|e^{i\chi(x)}$ with electromagnetic field:

$$\mathcal{D}^{(1)}_\nu \mathcal{D}^{(1)\nu} \mathcal{H} = \partial_{\mathcal{H}^*} V, \qquad (5)$$

$$\nabla_\nu \nabla^\nu A_\mu = I_\mu = \frac{1}{2} e|\mathcal{H}|^2 (\chi,_\mu + eA_\mu). \qquad (6)$$

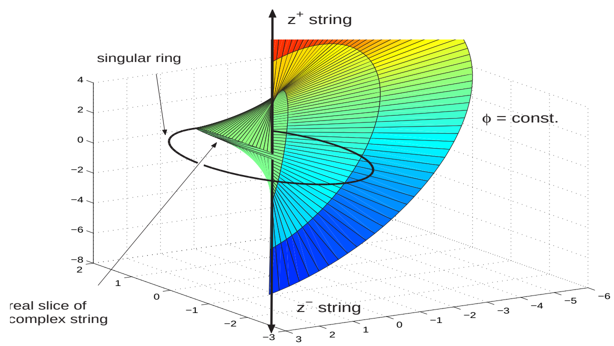


Figure 3: The Kerr surface $\phi = const$. The Kerr congruence is tangent to singular ring at $\theta = \pi/2$.

***Peculiarities of the KN soliton model*:**

(i) closed flux of the KN electromagnetic potential forms a *quantum Wilson loop* $\oint eA_\varphi d\varphi = -4\pi ma$, which results in quantization of the soliton spin, $J = ma = n\hbar/2, \ n = 1, 2, 3, ...$

(ii) the Higgs condensate forms a *coherent vacuum state* oscillating with the frequency $\omega = 2m$ – oscillons, Q-balls (G.Rosen 1968, Coleman 1985).

Supersymmety and Bogomolnyi bound. Hamiltonian:

$$H^{(ch)} = T_0{}^{0(ch)} = \frac{1}{2}\sum_{i=1}^{3}\ [\sum_{\mu=0}^{3}|\mathcal{D}_\mu^{(i)}\Phi^i|^2 + |\partial_i W|^2].$$

Kerr's coordinate system $x + iy = (r + ia)e^{i\phi}\sin\theta, \quad z = r\cos\theta, \quad t = \rho - r.$ **Vector potential**

$$A_\mu dx^\mu = -Re\ [(\frac{e}{r + ia\cos\theta})](dr - dt - a\sin^2\theta d\phi). \tag{7}$$

Terms $A_\phi d\phi$ **and** $A_t dt$ **drop out of the Hamiltonian due the constraints**

$$\mathcal{D}_t^{(1)}\Phi^1 = 0, \quad \mathcal{D}_\phi^{(1)}\Phi^1 = 0, \tag{8}$$

consistent with (i) and (ii). The rest is reduced to integral over variable r.

$$H^{(ch)} = T_0{}^{0(ch)} = \frac{1}{2}\sum_{i=1}^{3}\ [|\mathcal{D}_r^{(i)}\Phi^i|^2 + |\partial_i W|^2], \tag{9}$$

Then we use the TRICK suggested by Cvetiĉ & Rey for planar Dom Wall, which WORKS! and allows to transform Hamiltonian to Bogomolnyi form

$$H^{(ch)} = T_0{}^{0(ch)} = \frac{1}{2}\sum_{i=1}^{3}\ [|\mathcal{D}_r^{(i)}\Phi^i - e^{i\chi_i}\partial_i\bar{W}|^2 + 2Re\ e^{-i\chi_i}\partial_i\bar{W}\mathcal{D}_r^{(i)}\Phi^i] \tag{10}$$

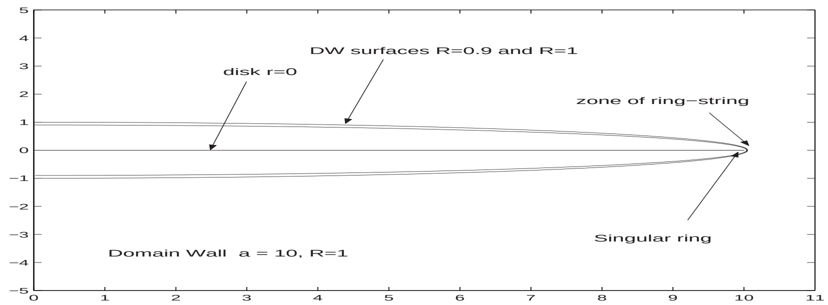


Figure 4: **Axial section of the spheroidal domain wall phase transition.**

The angles χ_i are determined by phase of the oscillating Higgs field

$$\Phi(x) \equiv \Phi^1(x) = |\Phi^1(r)| e^{i\chi(t,\phi)}. \tag{11}$$

It yields $\chi_1 = 2\chi(t,\phi)$, $\chi_2 = \chi_3 = 0$, and We obtain the Bogomolnyi equations

$$\mathcal{D}_r^{(i)} \Phi^i = \partial W / \partial \Phi^i, \quad \mathcal{D}_r^{(i)} \bar{\Phi}^i = \partial \bar{W} / \partial \bar{\Phi}^i. \tag{12}$$

Hamiltonian turns into full differential ($\mathcal{D}_r \to \partial_r$ due structure of W)

$$H^{(ch-r)} = Re \ (\partial W / \partial \Phi^i) \partial_r \Phi^i = \partial W / \partial r. \tag{13}$$

Using the Kerr coordinate system, and $\Delta W = W(R+\delta) - W(R-\delta) = -\mu\eta^2$, we obtain

$$\delta M_{bag} = 2\pi \Delta W \int_{-1}^{1} dX (R^2 + a^2 X^2) = 4\pi (R^2 + \frac{1}{3} a^2) \Delta W. \tag{14}$$

BPS-saturated soliton $\Rightarrow$ Stability.

KN source as a BAG model. Basic features:

- **Nonperturbative solution for vev based on the Higgs mechanism.**
- **Consistent embedding of the Dirac equation.**
- **Compliance to deformations and the formation of stringy structure.**
- **Mass of the Dirac field is position-dependent and generated by the Higgs field.**

Algebraically special KN solution – all fields are collinear to Principal Null Directions k^μ **of the Kerr congruence** k^μ.
Metric of the Kerr-Newman solution: $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$ **and electromagnetic vector potential** $A^\mu_{KN} = Re\frac{e}{r+ia\cos\theta}k^\mu$ **and also the Dirac field !!!.**
Two solutions of the Kerr theorem $Y^\pm(x)$ **determine two Weyl spinors.**

THE KERR THEOREM: Kerr congruence has two solutions $k^\pm_\mu$ **creating two metrics** $g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2Hk^\pm_\mu k^\pm_\nu$. **TWOSHEETED Kerr space!**

Geodesic and Shear-free congruences are obtained as analytic solutions of the equation $F(T^a) = 0$, ***where*** F ***is a holomorphic function of the projective twistor coordinates in CP***^{3}, $T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}$.
$Y^+ = \phi_1/\phi_0$, **is equivalent to Weyl spinor** ϕ_α **and** Y^-, **to** $\bar{\chi}^{\dot{\alpha}}$.

TWISTOR $\Leftrightarrow$ SPINOR relation is origin of the consistent Dirac field.

FERMIONIC SECTOR DIRAC EQUATION splits in the Weyl representation into two equations

$$\sigma^{\mu}_{\alpha\dot{\alpha}} i\partial_\mu \bar{\chi}^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = m\bar{\chi}^{\dot{\alpha}}, \tag{15}$$

the "left-handed" and "right-handed" electron fields, Weyl spinors.

Two antipodally conjugate solutions of the Kerr theorem $Y^+ = -1/\bar{Y}^-$ determine two Weyl spinor fields ϕ^α and $\bar{\chi}_{\dot{\alpha}}$, corresponding to antipodal congruences $Y^+ = \phi_1/\phi_0 \quad , \quad Y^- = \bar{\chi}^{\dot{1}}/\bar{\chi}^{\dot{0}}$
For Y^+ we have

$$\phi_\alpha = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \tag{16}$$

and for $Y^- = -1/\bar{Y}^+$,

$$\bar{\chi}^{\dot{\alpha}} = \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}. \tag{17}$$

The corresponding null vector fields $k^{\mu\pm}(x)$ differ on the retarded and advanced sheets, and generate different metrics

$$g^{\pm}_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)} k^{\pm}_\mu k^{\pm}_\nu. \tag{18}$$

The "left" and "right" Weyl components of the Dirac fields should be positioned on SEPARATE SHEETS of the Kerr space-time.

This requirement disappears inside the bag, where the space is flat, and both congruences $k^\pm_\mu(x)$ are consistent with the flat Minkowski space.

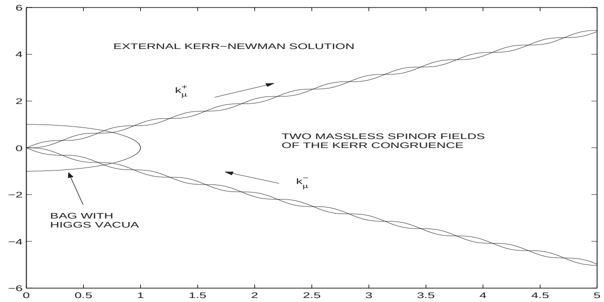


Figure 5: **Spinors ϕ_α $\bar\chi^{\dot\alpha}$ are placed on different sheets controlled by antipodal Kerr congruences. Inside the bag these spinor fields acquire Yukawa coupling giving mass to the Dirac equation.**

Inside the bag the massless Weyl spinors are united into a Dirac bispinor Ψ, and the Dirac equation $(\gamma^\mu\partial_\mu + m)\Psi(x) = 0$, acquires the mass $m(x) \equiv g\mathcal{H}(x)$ from the Higgs condensate $\mathcal{H}(x)$.

The position-dependent mass term is a feature of the BAG models!

== Link to QED. Remarkable properties of *supersymmetry* allows us to understand the connection with QED. Perturbative approach for supersymmetric field theory is developed as a direct extension of ordinary perturbation theory and Feynman rules are stated in terms of superfield vertices and propagators which have miraculous cancelations between components of the superfield diagram. In particular, all contributions to mass renormalization, cancel between the various component fields.

The Kerr singular ring forms a closed circular string ("gravitational waveguide" A.B.ZhETF 1974; Ivanenko & Burinskii, 1975; "fundamental solitonic string" A.Sen 1995, A.B.1995).

The EM field is concentrated on the sharp border of the bag, forming a *frozen* traveling wave (quasi-plane pp-wave, A.B. 1974). After regularization the Kerr singular ring-string is regularized with the cut-off parameter $R = r_e$.

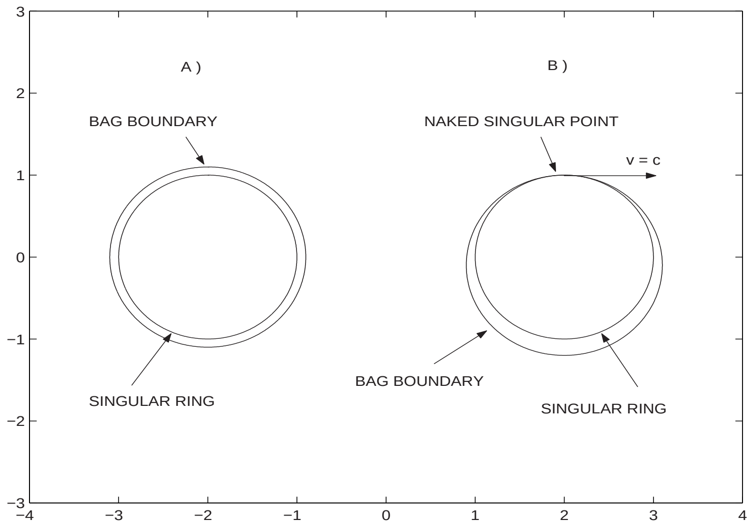
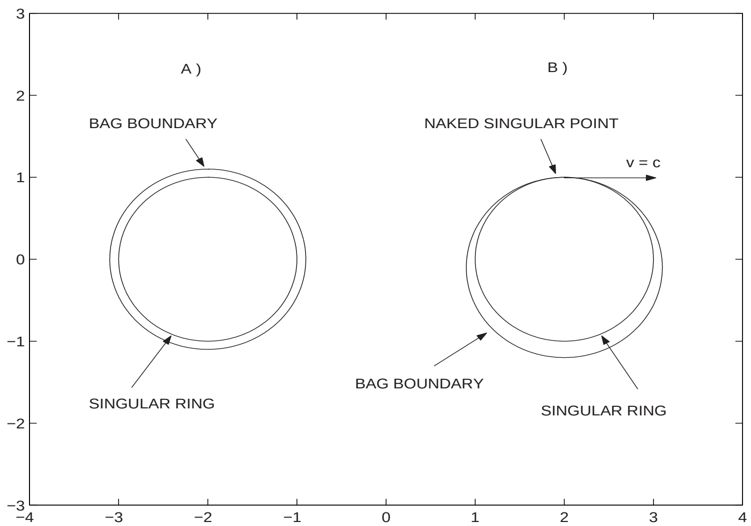


Figure 6: **Regularization of the string. Section of the disk-like bag in equatorial plane. Distance from boundary of the bag till position of the (former) singular ring acts as a cut-off parameter R. (A)Axially symmetric KN solution yields a constant cut-off $R = r_e$. (B)Traveling wave deforms the bag, creating singular point with (zitterbewegung)**.

The COMPTON radius of the Kerr-Newman bag corresponds to a DRESSED electron!

Does the Kerr-Newman electron contradict to the point-like electron of Quantum Theory and to the Experimental Data?

Any external observer will see the point-like Kerr-Newman source!
The light-like closed pp-string shrinks to point by Lorentz contraction! (Arcos & Pereira, 2006, A.B. 2009.)



Traveling circular pp-wave wave deforms the bag and creates a circulating singular pole leading to a BAG-STRING-QUARK system with ZITTERBEWEGUNG (A.B. ZhETF, 148 (4), 2015 (in press)).

INTERPLAY OF THE MASSIVE AND MASSLESS SECTORS

Integration of the Einstein-Maxwell field equations, (Debney, Kerr and Schild, JMP 1969).

Treatment of the generating function of the Kerr theorem F as a product of partial functions F_i for k spinning particles $F = \prod_i^k F_i, \ \ i = 1, ...k,$ leads to multi-sheeted, multi-twistorial space-time over M^4.

Structures of the i-th and j-th particles do not feel each other, forming their own internal space, A.B.***Grav. and Cosmol***, 11, 301 (2005).

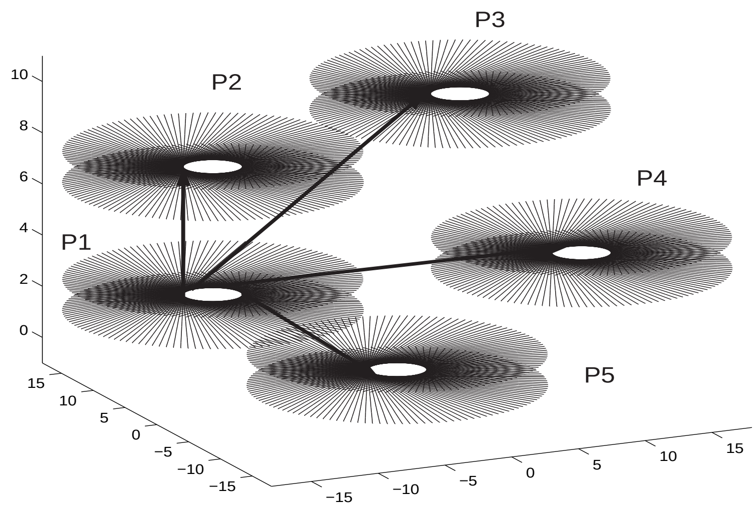


Figure 7: **Multi-sheeted twistor space. Each particle has twofold structure. Any excitations of the KS geometry creates a twistor-beam in the form of pp-wave solution.**

Interaction between particles occurs via light-like singular twistor lines, pp-wave beams, creating LIGHTLIKE network of the Kerr-Schild space-time.

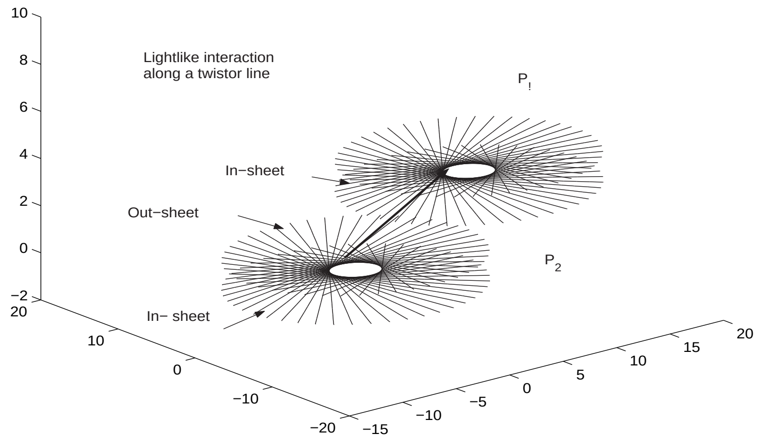


Figure 8: **Lightlike interaction via two common twistor lines connecting out-sheet of one particle to in-sheet of another.**

The pp-waves form fundamental string solutions to the low energy string theory (A. Sen, 1995), for which all quantum corrections vanish (G. T. Horowitz and A. R. Steif, PRL(1990)).

Each massive particle is formed as a bundle of twistors. Each two massive particles are joined by two common twistor lines, supporting two oppositionally directed pp-waves.

System of the exact Kerr-Schild solutions gives a network of the massive nodes connected by the light-like pp-wave strings.

NO MODIFICATION of the Einstein–Maxwell field equations!!!

Twistorial excitations of the Kerr-Schild space create the foam-like horizon of the Kerr black hole. (A.B. Theor.Math.Phys.163:782-787 (2010))

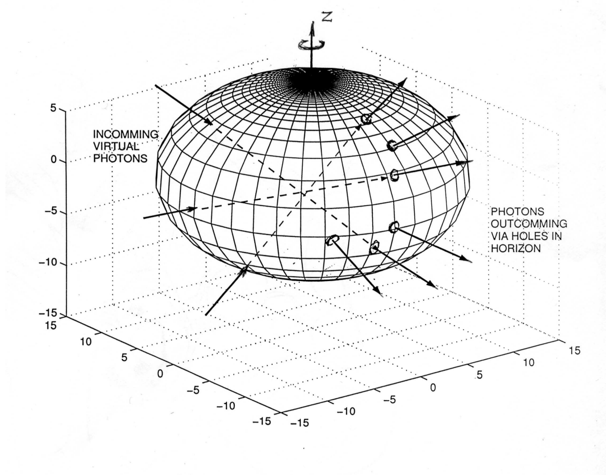


Figure 9: **The foam-like horizon. Excitations of a black hole by the field of virtual and real photons form a set of fluctuating micro-holes in horizon**.

CONCLUSION:

- **Source of the Kerr-Newman solution is obtained as a supersymmetric, BPS-saturated soliton of the disk-like form with quantum spin.**
- **SUPERSYMMETRY determines structure of the soliton, stability and vanishing of the quantum corrections.**
- ***Basic features of the Bag model,* embedding of the Dirac equation with the position-dependent mass determined by the Higgs condensate.**
- **The lowest excitation of the KN string (circular traveling pp-wave) create a circulating singular pole. The KN source forms a single Bag-String-Quark system.**
- **The multiparticle KS solutions form a network of the massive nodes connected by the light-like pp-waves strings.**
- **All quantum corrections vanish without MODIFICATION of the Einstein–Maxwell field equations!**

THANK YOU FOR ATTENTION!