Effective Quantum Gravity
and
Applications in Cosmology

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Outline

- Progress in Effective Quantum Gravity
- Applications to models of inflation (including Higgs inflation, Starobinsky's model)
- Applications to gravitational waves
- Conclusions

Perturbative linearized general relativity

Matter coupled to gravity described by general relativity:

$$S[g,\phi,\psi,A_\mu] \;= -\int d^4x\sqrt{-\det(g)}\;\left(\frac{1}{16\pi G_N}R+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\xi R\phi^2+ \right.$$
$$\left. +e\bar{\psi}i\gamma^\mu D_\mu\psi+\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

Linearized general relativity can be regarded as an effective field theory valid up to the reduced Planck mass

$$L=-\frac{1}{4}h^{\mu\nu}\Box h_{\mu\nu}+\frac{1}{4}h\Box h-\frac{1}{2}h^{\mu\nu}\partial_\mu\partial_\nu h+\frac{1}{2}h^{\mu\nu}\partial_\mu\partial_\alpha h^\alpha_\nu-\frac{\sqrt{2}}{\bar{M}_P}h^{\mu\nu}T_{\mu\nu}+\mathcal{O}(\bar{M}_P^{-2})$$

The theory is non-renormalizable, but some predictions are still possible.

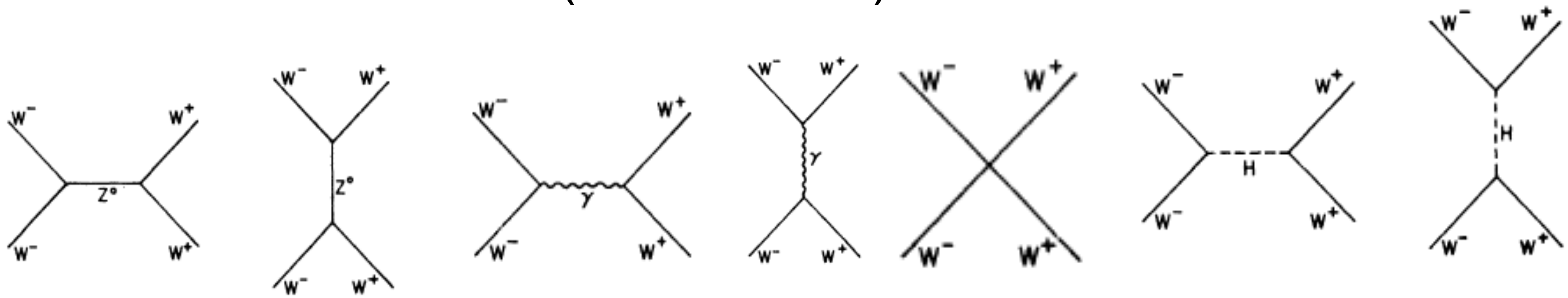
Unitary in perturbative quantum field theory

- Follows from the conservation of probability in quantum mechanics.

- Implies that amplitudes do not grow with energy.

- One of the few theoretical tools in quantum field theory to get information about the parameters of the model.

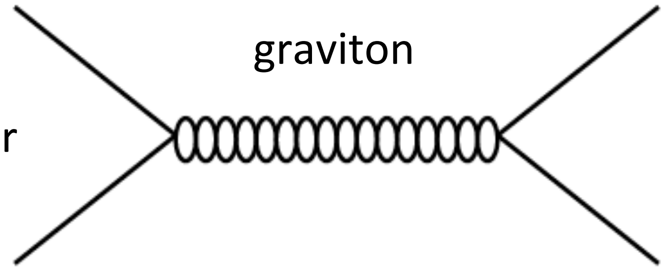
- Well known example is the bound on the Higgs boson’s mass in the Standard Model (m<790 GeV).



At what energy scale does the EQG breakdown?

Let us consider gravitational scattering of the particles included in that model (s-channel, we impose different in and out states)

(Han & Willenbrock 2004, xc & Atkins 2011)



$\to$	$s'\bar{s}'$	$\psi'_+\bar{\psi}'_-$	$\psi'_-\bar{\psi}'_+$	$V'_+V'_-$	$V'_-V'_+$
$s\bar{s}$	$-2\pi G_N s(1/3d^2_{0,0} - 1/3(1+12\xi)^2 d^0_{0,0})$	$-2\pi G_N s\sqrt{1/3}\ d^2_{0,1}$	$-2\pi G_N s\sqrt{1/3}\ d^2_{0,-1}$	$-4\pi G_N s\sqrt{1/3}\ d^2_{0,2}$	$-4\pi G_N s\sqrt{1/3}\ d^2_{0,-2}$
$\psi_+\bar{\psi}_-$	$-2\pi G_N s\sqrt{1/3}\ d^2_{1,0}$	$-2\pi G_N s d^2_{1,1}$	$-2\pi G_N s d^2_{1,-1}$	$-4\pi G_N s\ d^2_{1,2}$	$-4\pi G_N s\ d^2_{1,-2}$
$\psi_-\bar{\psi}_+$	$-2\pi G_N s\sqrt{1/3}\ d^2_{-1,0}$	$-2\pi G_N s d^2_{-1,1}$	$-2\pi G_N s d^2_{-1,-1}$	$-4\pi G_N s 2\ d^2_{-1,2}$	$-4\pi G_N s 2\ d^2_{-1,-2}$
V_+V_-	$-4\pi G_N s\sqrt{1/3}\ d^2_{2,0}$	$-4\pi G_N s\ d^2_{2,1}$	$-4\pi G_N s\ d^2_{2,-1}$	$-8\pi G_N s\ d^2_{2,2}$	$-8\pi G_N s\ d^2_{2,-2}$
V_-V_+	$-4\pi G_N s\sqrt{1/3}\ d^2_{-2,0}$	$-4\pi G_N s\ d^2_{-2,1}$	$-4\pi G_N s\ d^2_{-2,-1}$	$-8\pi G_N s\ d^2_{-2,2}$	$-8\pi G_N s\ d^2_{-2,-2}$

$$|\text{Re}\ a_J| \leq 1/2 \qquad \mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'}$$

At what energy scale does the EQG breakdown?

Let us look at J=2 partial wave

$$a_2 = -\frac{1}{320\pi}\frac{s}{\bar{M}_P^2}N \qquad N = 1/3N_s + N_\psi + 4N_V$$

One gets the bound:

$$E^*_{\rm CM} \leq \bar{M}_P\sqrt{\frac{160\pi}{N}}$$

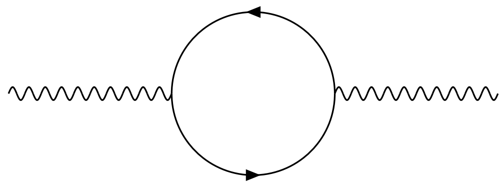
For large *N*, unitarity can be violated ***well below*** the Planck mass. From the J=0 partial wave, one gets

$$\Lambda \simeq \bar{M}_P/\xi$$

What is going on?

Self-healing of unitarity

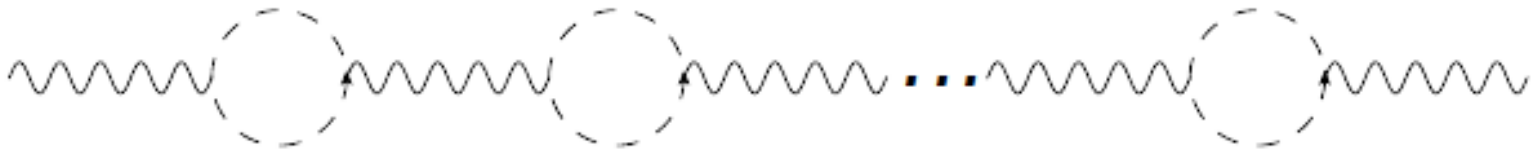
- Aydemir, Anber & Donoghue argued that the effective theory heals itself.
- First let's calculate the leading quantum corrections to the previous amplitude (still working in linearized GR in flat space-time)



- Insert any matter in your model in that loop (gravitons are suppressed, but can be included).
- Typically there is more matter than gravitational degrees of freedom, we can thus ignore gravitons in that loops for energies below the Planck mass.
- Honest calculation: regularized using dim-reg and absorb divergencies in R^2 etc.
- Obviously the theory is still not renormalizable, but that's not an issue for an effective field theory.

Self-healing of unitarity

- In the case of linearized gravity coupled to the SM, resum:



- in the large N limit, keeping NG_N small. One obtains a resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

$$N = N_s + 3N_f + 12N_V$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2$$

- One can check explicitly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

Self-healing of unitarity non-minimal coupling

- One can also resum the infinite series of 1-loop polarization diagrams

- In the large ξ and N limits but keeping N ξG_N small, I get

$$iD^{\alpha\beta\mu\nu}_{dressed} = -\frac{i}{2s}\frac{L^{\alpha\beta}L^{\mu\nu}}{\left(1-\frac{sF_1(s)}{2}\right)}$$

$$F_1(q^2) = -N_s G_N \xi^2 \log\left(\frac{-q^2}{\mu^2}\right)$$

- The dressed amplitude fulfills exactly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

XC & Casadio 2014

Self-healing yes, but…

- In linearized GR, the effective theory self-heals itself.
- In the large N limit keeping N G_N small one finds poles in the resummed graviton propagator: sign of strong interaction.
- The positions of these poles depend on the number of fields
- One finds

$$
\begin{aligned}
q_1^2 &= 0, \\
q_2^2 &= \frac{1}{G_N N} \frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}, \\
q_3^2 &= (q_2^2)^*,
\end{aligned}
$$

- Complex pole: EQG breaks down and potentially well below the Planck scale.

Poles and Quantum Black Holes?

- It is tempting to interpret these poles as black hole precursors.
- In the SM

$$N_s = 4,\ N_f = 45,\ \text{and}\ N_V = 12$$

- We thus find

$$(7 - 3i) \times 10^{18}\ \text{GeV}\ \text{and}\ (7 + 3i) \times 10^{18}\ \text{GeV}$$

- Using

$$p_0^2 = (m - i\Gamma/2)^2$$

- The first one corresponds to a state with mass

$$7 \times 10^{18}\ \text{GeV}$$

and width

$$6 \times 10^{18}\ \text{GeV}$$

- Note that the 2nd pole has the wrong sign for particle: it is a ghost

Analogy to QCD

- Our interpretation is similar to the sigma-meson case which can be identified as the pole of a resummed scattering amplitude in the large N limit of chiral perturbation theory.

- This resummed amplitude is an example of self-healing in chiral perturbation theory.

- In low energy QCD (chiral perturbation theory), the position of the pole does correspond to the correct value of the mass and width of the sigma-meson.

Acausal versus nonlocal effects

- Remember that the 2nd pole has the wrong sign between the mass and width terms for a particle: it is a ghost.

- Acausal effects: connection to black hole information paradox? Could be canceled by e.g. Lee and Wick's mechanism.

- Acausal effects can be replaced by non local effects

$$S = \int d^4x \sqrt{g} \left[R \log\left(\frac{\Box}{\mu^2}\right) R \right] \qquad L(x,y) = \langle x| \log\left(\frac{\Box}{\mu^2}\right) |y\rangle$$

by reinterpreting the log term (more later).

- Can these effects soften singularities?

Self-healing & Classicalization

- With our interpretation in mind, an interesting picture emerges.
- Self-healing in the case of gravitational interactions implies unitarization of quantum amplitudes via quantum black holes.
- As the center of mass energy increases so does the mass of the black hole and it becomes more and more classical.
- This is nothing but classicalization.
- What we call Planck scale (first QBH mass/cut off for the EFT) is now a dynamical quantity which depends on the number of fields.
- The effective theory certainly breaks down at the Planck scale.
- Self-healing makes the link between several concepts that had been proposed previously.

Once again perturbative unitarity

- Let's think about perturbative unitarity again.
- We are taught that a breakdown of perturbative unitarity is a sign of new physics or strong dynamics.
- In the case of quantum gravity in the large N, we have identified the strong dynamics as quantum black holes: this is not a surprise.
- More surprising is the case of a large nonminimal coupling of scalars to R, here we found a resummed propagator that does not have poles beyond the one at $q^2=0$.
- Unitarity is restored by the self-healing mechanism without new physics or strong dynamics.

Effective Quantum Gravity

- We thus have an EFT valid up to a scale $M_\star \sim \sqrt{\frac{120\pi}{NG_N}}$

- The leading order terms are

$$S = \int d^4x \sqrt{-g}\Big(\Big(\frac{1}{2}M^2 + \xi H^\dagger H\Big) R - \Lambda_C^4 + c_1 R^2 + c_2 C^2 + c_3 E + c_4 \Box R + \\ -L_{SM} - L_{DM} + O(M_\star^{-2})\Big)$$

$$E = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$C^2 = E + 2R_{\mu\nu}R^{\mu\nu} - 2/3R^2$$

NB the Wilson coefficients
of these operators must
Be measured in experiments.

Predictions of EFT

- The Wilson coefficients of these operators are predictions of quantum gravity.

$$S_{QL} = \int d^4x \sqrt{g} \left(\alpha R \log\left(\frac{\Box}{\mu_\alpha^2}\right) R + \beta R_{\mu\nu} \log\left(\frac{\Box}{\mu_\beta^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta} \log\left(\frac{\Box}{\mu_\gamma^2}\right) R^{\mu\nu\alpha\beta} \right)$$

	α	β	γ
Scalar	$5(6\xi - 1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Graviton	430	-1444	424

All numbers should be divided by $11520\pi^2$.

NB: they are calculated using dim-reg.

(Donoghue et al, Codello et al.)

- These operators correspond to the resummed graviton propagator we have considered and will lead to some non-local effects.

Summary of EQG and bounds on its parameters

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S=\int d^4x\,\sqrt{-g}\left[\left(\frac{1}{2}M^2+\xi H^\dagger H\right)\mathcal{R}-\Lambda_C^4+c_1\mathcal{R}^2+c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}+\mathcal{L}_{SM}+\mathcal{O}(M_\star^{-2})\right]$$

- Planck scale $(M^2+\xi v^2)=M_P^2$ $M_P=2.4335\times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1=9.7\times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6\times 10^{15}$ [xc & Atkins, 2013]

    Higgs inflation requires $\xi \sim 10^4$.

Applications to Cosmology

- Can EQG be probed in the CMB? Are there new signatures of this non-locality?
- Now that we have a consistent approach to quantum gravity, can we build new models of inflation based on EQG?
- Is there any effect in gravitational waves?

Can EQG be probed in the CMB?

XC, Croon & Fritz (2015)

- Gravity leads to non-local effects in Matter
- Let's reconsider the resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

- Using this propagator we can now calculate the dressed amplitude for the gravitational scattering of 2 scalar fields.
- The tree-level amplitude has been known for a long time:

$$\begin{aligned} A_{tree} \quad = \quad & 16\pi G\left(m^4\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right)+\frac{1}{2s}(2m^2+t)(2m^2+u)\right. \\ & \left.+\frac{1}{2t}(2m^2+s)(2m^2+u)+\frac{1}{2u}(2m^2+s)(2m^2+t)\right) \end{aligned}$$

Non-local effects in matter

- Let me rewrite the dressed propagator as

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{P^{\alpha\beta,\mu\nu}(q^2)}{1+f(q^2)}, \qquad f(q^2) = -\frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right).$$

- We find the Taylor expanded dressed amplitude:

$$A_{dressed} = A_{tree} + A^{(1)} + \dots$$

$$\begin{aligned} A^{(1)} = \frac{2}{15}G_N^2 N\Bigg(m^4\left(\log\left(-\frac{stu}{\mu^6}\right)\right) &+ \log\left(-\frac{s}{\mu^2}\right)(2m^2+t)(2m^2+u) + \log\left(-\frac{t}{\mu^2}\right)(2m^2+s)(2m^2+u) \\ &+ \log\left(-\frac{u}{\mu^2}\right)(2m^2+s)(2m^2+t)\Bigg). \end{aligned}$$

Higher order non-local operator

- It is easy to see that $A^{(1)}$ can be obtained from this effective operator:

$$O_8 = \frac{2}{15} G_N^2 N \left(\partial_\mu \phi(x) \partial^\mu \phi(x) - m^2 \phi(x)^2\right) \log\left(-\frac{\Box}{\mu^2}\right) \left(\partial_\nu \phi(x) \partial^\nu \phi(x) - m^2 \phi(x)^2\right)$$

- This is a non-local operator, we need to make sense of the log term to obtain a causal theory (Espiru et al. (2005), Donoghue &El-Menoufi (2014) and Barvinsky et al in the 80's.)

$$\begin{aligned} S &= \int d^4x d^4y \sqrt{-g} \left(\frac{1}{16\pi G_N} R - \frac{1}{2} \partial_\mu \phi(x) \partial^\nu \phi(x) + \frac{m^2}{2} \phi^2 \right. \\ &\quad + \left(\frac{2}{15} G_N^2 N \right) \times \end{aligned}$$

$$\left. \left(\left(\partial_\mu \phi(x) \partial^\mu \phi(x) + m^2 \phi(x)^2 \right) \int d^4y \sqrt{-g(y)} \langle x | \log\left(-\frac{\Box}{\mu^2}\right) | y \rangle \left(\partial_\nu \phi(y) \partial^\nu \phi(y) - m^2 \phi(y)^2 \right) \right) \right)$$

Non-local function

- One can define the interpolating function:

$$\mathcal{L}(x,y) = \langle x| \log\left(-\frac{\Box}{\mu^2}\right)|y\rangle$$

- which can be evaluated $\log(x) \approx -1/\epsilon + x^\epsilon/\epsilon$

$$\begin{aligned} -\langle x|\frac{1}{\epsilon}|y\rangle + \langle x|\frac{(\Box/\mu^2)^\epsilon}{\epsilon}|y\rangle &= -\frac{1}{\epsilon}\delta(x-y) + \frac{1}{\epsilon}\frac{2\pi^2}{\mu^{2\epsilon}}\int d^4k k^{2+2\epsilon}\frac{1}{|x-y|}J_1(k|x-y|) \\ &\sim -\frac{1}{\epsilon}\delta(x-y) - \frac{8\pi^2}{\mu^{2\epsilon}}\frac{1}{|x-y|^{4+2\epsilon}}, \end{aligned}$$

- For a purely time-dependent problem one has

$$\mathcal{L}(t,t') = -2\lim_{\epsilon\to 0}\left(\frac{\Theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')(\log(\mu\epsilon) + \gamma\right)$$

Gravity leads to non-local effects in Matter

- We have seen that the non-local effects observed in gravity feed back into matter.
- This is compatible with our interpretation of the poles of the resummed propagators as quantum black holes (black hole precursors) which are extended objects.
- The new higher dimensional operators have an approximate shift symmetry

$$\phi \to \phi + c, \text{ where } c \text{ is a constant}$$

- which is broken explicitly by the mass of the scalar field.
- This is interesting for models of inflation.

Non-gaussianities in single field inflation models

- Are there any observational consequences of this short distance non-locality?
- The effect is suppressed by powers of the Planck scale, one can see that it leads to a small non-Gaussianities even for a single scalar inflation model.
- However the effect is too small to be observable.
- Let's considering the following Lagrangian

$$
\begin{aligned}
L(x) &= X + \frac{m^2}{2}\phi^2(x) + \\
&+ \frac{8}{15} G_N^2 N \left(X(x) + \frac{m^2}{2}\phi^2(x) \right) \int d^4y \sqrt{-g(y)} \mathcal{L}(x,y) \left(X(y) + \frac{m^2}{2}\phi^2(y) \right)
\end{aligned}
$$

$$X(x) = -1/2 \partial_\mu \phi(x) \partial^\mu \phi(x) \qquad X(y) = -1/2 \partial_\mu \phi(y) \partial^\mu \phi(y)$$

Speed of sound

- We can calculate the speed of sound:

$$c_s^2 = \frac{L(x)_{,X(x)}}{L(x)_{,X(x)} + 2X(x)L(x)_{,X(x)X(x)}} \approx 1 - \frac{32}{15}X(x)G_N^2 N$$

- which remarkably to leading order does not depend on the specific form of the nonlocal function.

- GR coupled to a single scalar field thus predict a small amount from non-Gaussianity, but with a speed of sound very close to one.

- Non-locality is a generic feature of quantum field theory coupled to GR.

EQG and minimalistic inflation models

- Besides the 750 GeV events at CERN, there are not many signs of new physics beyond the standard model.
- It is still crucial to investigate whether the standard model Higgs and/or general relativity can describe inflation.
- EQG is the right framework for this.
- We need scalar degrees of freedom: Higgs boson or scalars hidden in higher gravitational operators such as R^2 (Starobinsky inflation).

EQG and minimalistic inflation models

- We have seen that a large non-minimal coupling of the Higgs boson to curvature does not introduce a new scale.

- An interesting possibility would be that a large non-minimal coupling of the Higgs boson to the Ricci scalar could lead to Starobinsky's R^2 inflation.

- Let me quickly first review R^2 inflation and Higgs inflation.

R^2 inflation

- The model is defined by the action in the Jordan frame

$$S^J_{Starobinsky} = \int d^4x \sqrt{g} \frac{1}{2} \left(M_P^2 R + c_S R^2 \right)$$

- which corresponds to an Einstein frame action given by

$$S^E_{Starobinsky} = \int d^4x \sqrt{g} \left(\frac{M_P^2}{2} R - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{M_P^4}{c_S} \left(1 - \exp\left(-\sqrt{\frac{2}{3}} \frac{\sigma}{M_P} \right) \right)^2 \right)$$

- Assuming that the scalar field σ hidden in R^2 takes large values in the early universe, a successful prediction of the density perturbation δϱ/ϱ requires

$$c_S = 0.97 \times 10^9$$

xc & Kuntz 2016

Higgs Starobinsky inflation

- Let us start from the action of EQG

$$S = \int d^4x \sqrt{-g}\left(\left(\frac{1}{2}M^2 + \xi H^\dagger H\right) R - \Lambda_C^4 + c_1 R^2 + c_2 C^2 + c_3 E + c_4 \Box R + -L_{SM} - L_{DM} + O(M_\star^{-2})\right)$$

- The running of c_1 is depend on the Higgs non-minimal coupling

$$\mu \partial_\mu c_1(\mu) = \frac{(1-12\xi)^2}{1152\pi^2} N_s$$

See Codello et al. Donoghue et al.

- We find

$$c_1(\mu_2) = c_1(\mu_1) + \frac{(1-12\xi)^2 N_s}{1152\pi^2} \log \frac{\mu_2}{\mu_1}$$

xc & Kuntz 2016

Higgs Starobinsky inflation

- The bounds on c_1 are very weak in today's universe ($c_1 < 10^{61}$)
- Even if c_1 is of order unity today it would have been much larger in the early universe if the Higgs boson non-minimal coupling was large.
- Indeed, we assume that inflation took place at some high energy scale e.g. 10^{15} GeV, the log term is a factor of order 60 if we take the scale μ_1 of the order of the cosmological constant.
- A Higgs non-minimal coupling to the Ricci scalar of $= 1.8\times10^4$ would lead to a coefficient $c_1 = 0.97\times10^9$ for R^2.
- Assuming that the scalar extra degree contained in R^2 took large field values in the early universe, a large non-minimal coupling of the Higgs boson to the Ricci scalar can trigger Starobinsky inflation even if the standard model vacuum is metastable as the Higgs boson itself does not roll down its potential during inflation.
- Inflation is due entirely to the R^2, but is triggered by the Higgs large non-minimal coupling.

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Is the potential stable?

- The effective action is in the early universe given by

$$S_{EFT} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R + \alpha R^2 + \beta R \log\frac{-\Box}{\mu^2}R + \gamma C^2 + \ldots\right)$$

- with $\alpha = c_1 \times 16\pi G$ $\gamma = c_2 \times 16\pi G$ $c_1 = 0.97 \times 10^9$

- The parameter β is a prediction (as explained before) of EQG

$$N_s(1-12\xi)^2/(2304\pi^2)\times 16\pi G$$

- and it is indeed large for a large Higgs non-minimal coupling

$$7.8 \times 10^6$$

- We need to check the effective potential carefully

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Is the potential stable?

- Note that the effective action has been fully regularized and renormalized in the Jordan frame where it was defined.

- We can thus treat the EFT as a classical theory of the type F(R) with

$$F(R) \;=\; R + \alpha R^2 + \beta R \log \tfrac{-\Box}{\mu^2} R$$

- and map it to the Einstein frame to study the scalar potential:

$$V(\phi) = \frac{1}{2\kappa^2}\left(e^{\sqrt{\frac{2}{3}}\kappa\phi} R(\phi) - e^{2\sqrt{\frac{2}{3}}\kappa\phi} F(R(\phi))\right)$$

where $\kappa^2 = 8\pi G$ and $R(\phi)$ is a solution to the equation

$$\phi = -\sqrt{\frac{3}{2}}\frac{1}{\kappa} \log \frac{dF(R)}{dR}.$$

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Is the potential stable?

- We can find a formal solution:

$$R(\phi) = \frac{1}{2\alpha}\left(\frac{1}{1+\frac{\beta}{2\alpha}\log\left(\frac{-\Box}{\mu^2}\right)}\right)\left(e^{-\sqrt{\frac{2}{3}}\kappa\phi}-1\right)$$

- We can be understood as a series in $\frac{\beta}{2\alpha} \sim 4\times 10^{-3}$

$$R(\phi) = \frac{1}{2\alpha}\left(1-\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{\beta}{2\alpha}\log\left(\frac{-\Box}{\mu^2}\right)\right)^n\right)\left(e^{-\sqrt{\frac{2}{3}}\kappa\phi}-1\right)$$

- To zeroth order we recover the usual R^2 model

$$R(\phi)^{(0)} = R(\phi)_{Starobinsky} = \frac{1}{2\alpha}\left(e^{-\sqrt{\frac{2}{3}}\kappa\phi}-1\right)$$

The potential is stable

- The series expansion will generate higher order terms corresponding to operators of the type

$$\exp(-\sqrt{\tfrac{2}{3}}\kappa\phi)(2/3\kappa^2\partial_\mu\phi\partial^\mu\phi - \sqrt{2/3}\kappa\Box\phi)$$

and higher derivatives thereof.

- These new terms are however suppressed by powers of

$$\frac{\beta}{2\alpha} \sim 4 \times 10^{-3}$$

and can be safely ignored.

- Note that log-term appearing in the F(R) term of the potential is also suppressed by β/2α compared to the usual R^2 potential.

Two comments about recent literature

- Herranen et al made two bold claims recently:
  - In Phys. Rev. Lett. 113, 211102 (2014): large fluctuations of order H in case of a high inflationary scale as suggested by BICEP2. They claim that for a high inflationary scale a large curvature mass is generated due to RG running of non-minimal coupling ξ, which either stabilizes the potential against fluctuations for $\xi \gtrsim 6 \cdot 10^{-2}$, or destabilizes it for $\xi \lesssim 2 \cdot 10^{-2}$ when the generated curvature mass is negative. Only in the narrow intermediate region the effect of the curvature mass may be significantly smaller
  - In Phys. Rev. Lett. 115, 241301 (2015): claiming find that for $\xi \gtrsim 1$, rapidly changing space-time curvature at the end of inflation leading to significant production of Higgs particles, potentially triggering a transition to a negative-energy Planck scale vacuum state and causing an immediate collapse of the Universe.

Two comments about recent literature

- Unfortunately, or rather fortunately for models of high scale inflation, these papers do not look quite right to me.
- In the first one they used an incorrect running for the non-minimal coupling of the Higgs field.
- Moss actually reached a different conclusion using a universal (and frame independent) beta-function (arXiv:1509.03554)
- The calculation of this beta-function is indeed tricky and has been confusing people for a while (the literature is full of conflicting results).

Two comments about recent literature

- In the second paper, they treated the term $R H^2$ term as a mass term for the Higgs boson.

- This is really too naive, as it is well known that this term leads to a mixing between the kinetic term of the Higgs boson and of the graviton and it needs to be diagonalized.

- In other words, the Higgs and graviton fields decouple and the non-minimal coupling does not contribute to the mass of the Higgs boson.

- There is no violation of the equivalence principle: the Higgs boson couples with the same strength as all other fields to gravity and it is not produced massively by the inflaton via this coupling during inflation.

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Gravitational Waves in Effective Quantum Gravity

- Let us go back once again to the resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

- where

$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2$ and μ is the renormalization scale

- From the resummed graviton propagator in momentum space, we can directly read o the classical field equation for the spin 2 gravitational wave in momentum space

$$2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right) = 0.$$

Gravitational Waves in Effective Quantum Gravity

- We have solved this equation earlier and found

$$
\begin{aligned}
q_1^2 &= 0, \\
q_2^2 &= \frac{1}{G_N N} \frac{120\pi}{W\left(\frac{-120\pi}{\mu^2 N G_N}\right)} \\
q_3^2 &= (q_2^2)^*,
\end{aligned}
$$

- The complex pole corresponds to a new massive degree of freedom with a complex mass (i.e. they have a width).

- The general wave solution is thus of the form

$$h^{\mu\nu}(x) = a_1^{\mu\nu} \exp(-iq_{1\alpha} x^\alpha) + a_2^{\mu\nu} \exp(-iq_{2\alpha} x^\alpha) + a_3^{\mu\nu} \exp(-iq_{2\alpha}^\star x^\alpha).$$

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Gravitational Waves in Effective Quantum Gravity

- We therefore have three degrees of freedom which can be excited in gravitational processes leading to the emission of gravitational waves.

- Note that our solution is linear, non-linearities in gravitational waves have previously been investigated and are as expected very small.

- To a very good approximation, we find the mass of the complex pole

$$m_2 = (0.53 - 0.67\, i)\sqrt{\frac{120\pi}{G_N N}}$$

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Gravitational Waves in Effective Quantum Gravity

- This excitation corresponds to a wave with the frequency

$$
\begin{aligned}
w_2 &= q_2^0 = \pm\sqrt{\vec{q}_2.\vec{q}_2 + (0.17 + 0.71\ i)\frac{120\pi}{G_N N}} \\
&= \pm\left(\frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}\right)^2 + \left(0.71\frac{120\pi}{G_N N}\right)^2} + \vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}}\right. \\
&\left. + i\frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\vec{q}_2.\vec{q}_2 + 0.17\frac{120\pi}{G_N N}\right)^2 + \left(0.71\frac{120\pi}{G_N N}\right)^2} - \vec{q}_2.\vec{q}_2 - 0.17\frac{120\pi}{G_N N}}\right).
\end{aligned}
$$

- The imaginary part of the complex pole will lead to a damping of the component of the gravitational wave corresponding to that mode.

Gravitational Waves in Effective Quantum Gravity

- The complex poles are gravitationally coupled to matter, thus the massive modes are produced at the same rate as the usual massless graviton mode if this is allowed kinematically.

- During an astrophysical event leading to gravitational waves, some of the energy will be emitted into these massive modes which will decay rather quickly because of their large decay width.

- The possible damping of the gravitational wave implies that care should be taken when relating the energy of the gravitational wave observed on earth to that of the astrophysical event as some of this energy could have been dissipated away as the wave travels towards earth.

Gravitational Waves in Effective Quantum Gravity

- Since the complex poles couple with the same coupling to matter as the usual massless graviton, we can think of them as a massive graviton although strictly speaking these objects have two polarizations only in contrast to massive gravitons that have five.

- This idea has been applied in the context of F(R) gravity arXiv: 1603.09551.

- We shall assume that these massive modes can be excited during the merger of two black holes.

- As a rough approximation, we shall assume that all the energy released during the merger is emitted into these modes.

- Given this assumption, we can use the limit derived by the LIGO collaboration on a graviton mass: $m_g < 1.2 \times 10^{-22}$ eV

Gravitational Waves in Effective Quantum Gravity

- We thus find a bound

$$\sqrt{\mathrm{Re}\left(\frac{1}{G_N N}\frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}\right)} < 1.2\times 10^{-22}\ \mathrm{eV}$$

- we thus obtain a lower bound on N: $N > 4\times10^{102}$ if all the energy of the merger was carried away by massive modes.
- Clearly this is not realistic as the massless mode will be excited.
- However, it implies that if the massive modes are produced, they will only arrive on earth if their masses are smaller than 1.2×10^{-22} eV.
- Waves corresponding to more massive poles will be damped before reaching earth.
- We shall see that there are tighter bounds on the mass of these objects coming from Eötvös type pendulum experiments.

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GW150914 and heavy waves?

- The LIGO collaboration estimates that the gravitational wave GW150914 is produced by the coalescence of two black holes: the black holes follow an inspiral orbit before merging, and subsequently going through a final black hole ringdown.
- Over 0.2 s, the signal increases in frequency and amplitude in about 8 cycles from 35 to 150 Hz, where the amplitude reaches a maximum.
- The typical energy of the gravitational wave is of the order of 150 Hz or 6×10^{-13} eV.
- In other words, if the gravitational wave had been emitted in the massive mode, they could not have been heavier than 6×10^{-22} GeV.
- However, this shows that it is perfectly conceivable that a sizable number of massive gravitons with $m_g < 6 \times 10^{-22}$ eV could have been produced.

Bounds from Eötvös type pendulum experiments

- We have seen that the resummed graviton propagator discussed above can be represented by the effective operator

$$\frac{N}{2304\pi^2} R \log\left(\frac{\Box}{\mu^2}\right) R$$

- The log term will be a contribution of order 1, this operator is thus very similar to the more familiar c R^2 term studied by Stelle long ago.

- The current bound on the Wilson coefficient of c is $c < 10^{61}$.

- We can translate this bound into a bound on N: $N < 2\times10^{65}$.

- This implies that the mass of the complex pole must be larger than 5 $\times10^{-13}$ GeV.

- This bound, although very weak, is more constraining than the one we have obtained from the graviton mass by 37 orders of magnitude.

Conclusions

- We have discussed a conservative effective action for quantum gravity (EQG) within usual QFTs such as the standard model.
- EQG can make predictions which can be confronted to data.
- One of the most exciting predictions is the existence of non-locality, in the form of new poles beyond the massless graviton.
- These poles can be interpreted as black hole precursors and their masses and widths can be calculated.
- These poles lead to non-local effects in QFT and in gravity.
- We have investigated models of inflation within EQG and found new connections between well known models of inflation.
- Finally we have shown that these poles could play an important role for gravitational waves.

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Thanks for your attention!