Particles with non-Abelian charges

Based on work done with N. Ahmadiniaz, F. Bastianelli, R. Bonezzi, J. Edwards, E. Latini, C. Schubert, A. Waldron

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II Flag Meeting "The Quantum and Gravity"
06 June 2016

Worldline methods: Quantum Field Theory results from qzn of QM models

- Main tools in use: particle actions

$$\text{(schematically)} \quad S[x,\psi;G] = \int_0^T d\tau \Big(\dot{x}^2 + \psi\dot{\psi} + V(x,\dot{x},\psi;G) \Big)$$

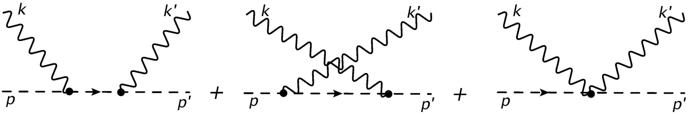
$$x \text{ bosonic} \quad \psi \text{ fermionic} \quad G \text{ external}$$

- canonical qzn
- particle path integrals:
  - topology of a circle: PBC $\longrightarrow$ one loop effective action

    review by Schubert '01
  - topology of a line: DBC $\longrightarrow$ dressed propagator, tree level amplitudes

Some Advantages of Worldline Formalism

- directly obtain off-shell Feynman amplitudes, rather than single Feynman diagrams. Ex: Compton scattering in scalar QED



- gauge-invariance efficiently guaranteed
- worldline formalism works well *also* with massive particle and at one loop
- present talk: tree-level results

Tree-level scalar QED

Coupling relativistic particle to external vector field

$$\langle \phi(x)\bar{\phi}(x')\rangle_A = \int_0^\infty dT\ e^{-Tm^2} \int_{x(0)=x'}^{x(T)=x} Dx\ e^{-S[x,A_\mu]}$$

$$S[x(\tau),A_\mu] = \int_0^1 d\tau\ \left(\frac{1}{4T}\delta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + iq\dot{x}^\mu A_\mu(x(\tau))\right)$$

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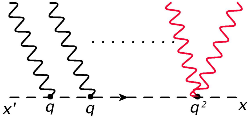
- treat A_μ perturbatively: get tree-level diagrams of scalar QED with 2 scalars and n photons

$$\langle \phi(x)\bar{\phi}(x')\rangle_A = \sum$$



- the WL linear coupling also reproduces the sea-gull coupling of QFT

Tree-level scalar QED - full momentum amplitude

- FT external lines to get full momentum amplitude

$$\tilde{\mathcal{A}}[p, p', k_1, \epsilon_1; \cdots; k_n, \epsilon_n] = q^n \int_0^\infty dT \, e^{-T(m^2+p^2)} \prod_{i=1}^n \int_0^1 d\tau_i$$

$$e^{T(p-p')\cdot\sum_i(-\tau_i k_i + i\epsilon_i)} e^{T\sum_{i,j}\left(k_i\cdot k_j \Delta_{ij} - i2\epsilon_i\cdot k_j \dot{\Delta}_{ij} + \epsilon_i\cdot\epsilon_j \ddot{\Delta}_{ij}\right)}\Big|_{m.l.\epsilon_i}$$

where $\Delta_{ij} = \frac{1}{2}|\tau_i - \tau_j|$

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- spinor QED amplitudes: matrix-valued spin factor or worldline supersymmetry (spinning particles) w.i.p. with Bastianelli, Edwards, Schubert, ...

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- In Scalar QCD $e^{-\int_0^1 d\tau ig\dot{x}\cdot W}$, is NOT gauge-invariant ($W_\mu = W_\mu^a T_a$)

  $\mathcal{P}$ path-ordering is needed $\mathcal{P}e^{-\int_0^1 d\tau\left(\frac{1}{4T}\dot{x}^2+ig\dot{x}\cdot W\right)}$

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- Alternatively: auxiliary fields can be used to generate color DoF's

Formalism

- non-abelian coupling $S_{int}[x; W] = ig \int d\tau \dot{x}^\mu W_\mu^a {(T_a)_\alpha}^{\alpha'}$ with $\alpha = 1, \ldots, N$ of $SU(N)$

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- kinetic term $S[c] = \int d\tau \bar{c}^\alpha \dot{c}_\alpha$
- canonical qzn $[c_\alpha, \bar{c}^{\alpha'}] = \delta_\alpha^{\alpha'}$ harmonic oscillators
- Fock space $|0\rangle,\ \bar{c}^\alpha|0\rangle, \bar{c}^{\alpha_1}\bar{c}^{\alpha_2}|0\rangle, \dots$
- representation is reducible: $\bullet + \square + \square\square + \cdots$

Formalism

- the particle action is invariant under a U(1) symmetry
  $c(\tau) \to e^{i\alpha} c(\tau)$, $\bar{c}(\tau) \to \bar{c}(\tau) e^{-i\alpha}$
- can restrict to a fixed occupation number
  (i) add a worldline gauge field $a(\tau)$, i.e. make the U(1) symmetry local Bastianelli, Bonezzi, OC, Latini 2013, 2015
  (ii) add a Chern Simons term

$$S[c; a] = i \int d\tau \, a(\bar{c}^{\alpha} c_{\alpha} - r)$$

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- At the quantum level:
  it fixes the occupation number, i.e. $\underbrace{\boxed{}\boxed{}\boxed{}\boxed{}}_{r}$

Scalar QCD propagator

- c, $\bar{c}$ correlation functions take care of the $\mathcal{P}$ ordering

$$\int \frac{DeDa}{\text{vol gauge}} \int_L DxD\bar{c}Dc\ e^{-S_t[x,c,w,a;W]}$$

- gauge field $a \xrightarrow{\text{gauge fix}}$ angle θ, $\quad e \xrightarrow{\text{gauge fix}} 2T$
- aux fields have as well Dirichlet b.c.'s: they carry the color states of the in and out scalars Ahmadiniaz, Bastianelli, OC, 2016

$$\langle\phi(x,\bar{u})\bar{\phi}(x',u')\rangle_W = \int_0^\infty dT\ e^{-Tm^2} \int_0^{2\pi} \frac{d\theta}{2\pi} e^{is\theta}$$
$$\int_{x(0)=x'}^{x(1)=x} Dx \int_{c(0)=u'}^{\bar{c}(1)=e^{-i\theta}\bar{u}} D\bar{c}Dc\ e^{-S_t[x,c,2T,\theta;W]+\bar{c}c(1)}$$

Tree-level amplitudes from Scalar QCD propagator

Recipe is the same as before: $W = \sum \text{gluons}$, multilinearity in ϵ's, Fourier transform the external scalar lines

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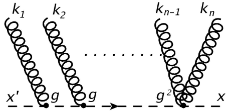
$$\mathcal{A}[p, u, p' u', k_1, \epsilon_1, a_1, ..., k_n, \epsilon_n, a_n]$$

$$= g^n \int_0^\infty dT\ e^{-Tm^2} \prod_{l=1}^{n} \int_0^1 d\tau_l$$

$$\times\ e^{T(p-p')\cdot\sum_l(-k_l\tau_l + i\epsilon_l)}\, e^{T\sum_{l,l'}(k_l\cdot k_{l'}\Delta_{ll'} - i2\epsilon_l\cdot k_{l'}\, {}^{\bullet}\Delta_{ll'} - \epsilon_l\cdot\epsilon_{l'}\, {}^{\bullet}\Delta^{\bullet}_{ll'})}\Big|_{m.l.\epsilon_l}$$

$$\times \int_0^{2\pi} \frac{d\theta}{2\pi} e^{ir\theta + e^{-i\theta}\bar{u}u'} \left\langle \prod_{l=1}^{n} (e^{-i\theta}\bar{u} + \kappa(\tau_l)) \cdot T^{a_l} \cdot (u + \kappa(\tau_l)) \right\rangle$$

$$= \sum$$



- The correlator of auxiliary fields gives the color factor

Summary

- Obtained several new applications of the method, at one-loop level and tree level, for QED, QCD
  - Auxiliary fields to generate path ordering and select arbitrary irrep of gauge group
  - Compact formula for n-gluon amplitudes
  - More flavors in aux fields with more CS charges: arbitrary YT

    OC, Edwards, 2016

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Outlook:

- Fields with spin at tree-level: (non)abelian spinning particles
- Study bound states. Done for scalar fields Bastianelli, Huet et al 2014
- Coupling to gravity (n. l. σ models)