How classical spacetimes could emerge from quantum gravity

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Formally quantize the system:

$$\hat{H} = \hat{H}_o \otimes \hat{I} + \frac{1}{2} \sum_k \left[\hat{I} \otimes \hat{\pi}_k^2 + k^2 \hat{a}^4 \otimes \hat{\phi}_k^2 \right]$$

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0th Born-Oppenheimer approximation:

$$\Psi(\tau, a, \phi) = \Psi_o(\tau, a) \otimes \varphi(\tau, \phi), \qquad i \frac{d}{d\tau} |\Psi_o\rangle = \hat{H}_o |\Psi_o\rangle$$

QFT on quantum spacetime:

$$i\frac{d}{d\tau}|\varphi\rangle = \frac{1}{2}\sum_k \left[1\hat{\pi}_k^2 + \langle\hat{a}^4\rangle k^2\hat{\phi}_k^2\right]|\varphi\rangle$$

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$$N = a^3, \qquad a = \langle\hat{a}^4\rangle^{\frac{1}{4}}$$

Interpreation: the dynamics of a massless quantum field ϕ on quantum spacetime Ψ_o is equivalent to the dynamics of ϕ on effective spacetime

$$ds^2 = -N^2 dt^2 + a^2 d\vec{x}^2 = -\langle \hat{a}^4 \rangle^{\frac{3}{2}} dt^2 + \langle \hat{a}^4 \rangle^{\frac{1}{2}} d\vec{x}^2$$

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Can we generalize?

Outline

1. generalization to massive case
2. applications to physical cosmology
3. contact with observations

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Two approaches:

1. put together second and third eq's (Assanioussi, AD, Lewandowski [1412.6000])
2. make the effective mass an unknown (Agullo, Ashtekar, Neslon [1211.1354])

approach 1: 2 equations for 2 unknowns

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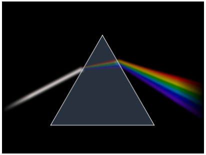
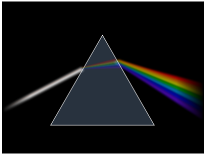
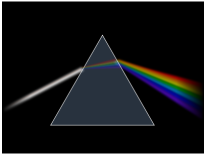
$$\bar{m} = \frac{\langle \hat{a}^6 \rangle^{\frac{1}{2}}}{\langle \hat{a}^4 \rangle^{\frac{3}{4}}} m$$

$\Rightarrow$ Effective mass is a renormalization of m by time-dependent multiplicative factor!





$$E^2 = m^2 + (1+\beta)P^2, \qquad \beta := \frac{\langle \Psi_o | \hat{a}^4 | \Psi_o \rangle}{\langle \Psi_o | \hat{a}^6 | \Psi_o \rangle^{\frac{2}{3}}} - 1$$



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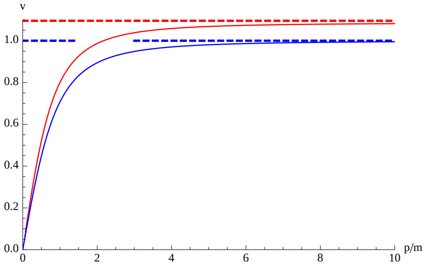
$\Rightarrow$ deformation controlled by parameter β of quantum gravity origin, and amounts to a renormalization of the speed of light: $c_{ren} = c\sqrt{1+\beta}$



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$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} m^2 \phi^2 \right)$$

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- tensor perturbations: q_m $(m = +, \times)$

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Physical Hamiltonian for matter and geometry:

$$H = H_o(a) + H_S(a,\phi) + \sum_m H_m(a, q_m)$$

where

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Up to H_o^{-1} factor, these are precisely the Hamiltonians of the toy model presented:

- ϕ is massive field
- q_+ and $q_\times$ are massless fields

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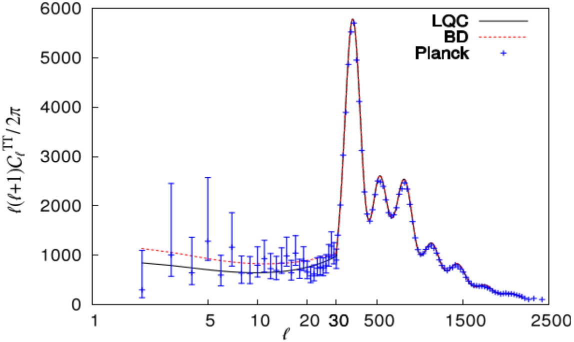
$\Rightarrow$ negligible effect today (in agreement with observations)

CMB power spectrum (Agullo, Ashtekar and Nelson [1302.0254]):



Predictions for the temperature-temperature correlation C_ℓ^{TT} obtained from LQC (black) and standard inflation (red). Both predictions are within the Planck observational data (blue error bars). The horizontal axis is enlarged for the $\ell < 30$ modes because both curves agree for $\ell > 30$.

conclusions

Done:

- mechanism: emergence of spacetime $g_{\mu\nu}$ from QFT on quantum cosmology
- Lorentz-violation controlled by "refractive index" β: a test for your favorite QG
- application to physical cosmology: LQG predictions in CMB power spectrum!

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To do:

- investigate early cosmology (Starobinsky inflation, non-gaussianities, ...)
- generalize the mechanism beyond cosmology (e.g., spherical collapse)

GRAZIE!