Thermodynamics of Local causal horizon for higher derivative gravity

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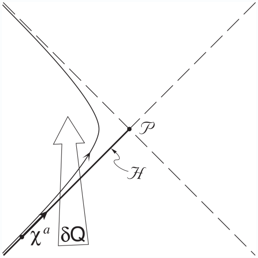
Outline

- Einstein equation from the Clausius relation
  Ted Jacobson, arXiv:gr-qc/9504004
- Defining a locally inertial coordinate system
  Raf Guedens, arXiv:1201.0542
- Black hole entropy as a Noether Charge
- Higher curvature equation of state and further problems
  Raf Guedens, Ted Jacobson, and Sudipta Sarkar, arXiv:1112.6215
- Proposal of a new entropy density
  Dey, Liberati and Mohd,arXiv:1605.04789
- Higher derivative equation of state(Local thermodynamics of such theories)

Einstein Equation of State

- Through 'p' we consider a small spacelike 2 surface element $\mathcal{P}$
- The past horizon of such a $\mathcal{P}$ is called the local Rindler horizon
- The causal horizon is associated with entropy becase they hide information. It can be thought of as the entanglement entropy which is proportional to the area of the horizon.
- Vacuum fluctions have a thermal nature when observed by an accelerated observer.
- Heat is defined as the energy that flows across a 'causal horizon'



Einstein Equation of State

- There is an approximate Killing vector, ξ^a, generating boosts in the region $\mathcal{P}$.
- According to Unruh effect T is given as $\hbar\kappa/2\pi$
- The heat flux is given as

$$\delta Q = \int T_{ab}\xi^a d\Sigma^b = -\kappa \int \lambda T_{ab}k^a k^b d\lambda dA$$

- The entropy change is given as

$$\delta A = \int \theta d\lambda dA = -\int \lambda R_{ab}k^a k^b d\lambda dA$$

Einstein Equation of State

Using Clausius relation, $\delta Q = T\delta S$ and assuming its validity at ever arbitrary spacetime point we get

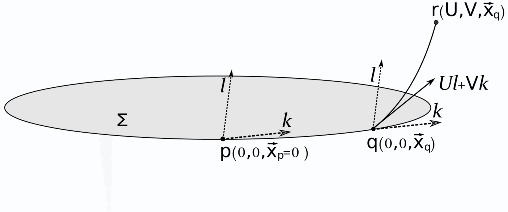
$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = \frac{2\pi}{\hbar\eta}T_{ab}$$

For arriving at this we used the conservation of the energy momentum tensor as a integrabilty condition for the left hand side of this equation.

Higher curvature gravity

- Previously the Entropy was chosen as the entanglement entropy of the system consisting of the local causal horizon as the boundary.
- For higher curvature theories it is well known that the black hole entropy is not simply proportional to the area of the horizon.
- Using the Noether charge entropy might be a good starting point.

Local inertial coordinates



- On Σ we pick RNC based on p
- Tangent space orthogonal to Σ spanned by null vectors k^a and l^a
- Each point r in neighbourhood of p lies on a unique geodesic passing through Σ
- The horizon is defined at $U = 0, V \leq 0$

Approximate Killing vector

A local Killing vector is assumed such that it has a bifurcation point on Σ. The Killing vector can be written as a power series in NNC and can shown to satisfy the following identity

$$\begin{aligned} \xi^{\mu}|_{\Gamma} &= (V - V_0)\,\delta^{\mu}_{V}, \\ \nabla_{(\mu}\xi_{\nu)} &= O(x^2), \\ \nabla_{\mu}\nabla_{\nu}\,\xi_{\rho}|_{\Gamma} &= (\,R_{\rho\nu\mu}{}^{\eta}\,\xi_{\eta})|_{\Gamma}\,. \end{aligned}$$

Noether charge entropy

- Wald proposed an expression of Blackhole entropy in terms of Noether Charge such that the first law holds for stationary Killing horizon.
- For a theory decribed by the Lagrangian

$$L = L\left[g_{ab}, R_{abcd}, \nabla_{a_1} R_{abcd}, ..., \nabla_{(a_1} ... \nabla_{a_m)} R_{abcd}, \psi, \nabla_{a_1}\psi, ..., \nabla_{(a_1} ... \nabla_{a_l)}\right.$$

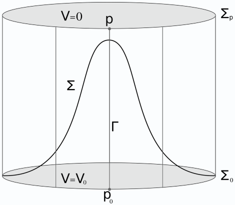
The Noether charge is given as

$$Q^{ab} = P^{abcd}\nabla_c \xi_d + W^{abc}\xi_c + Y^{ab} + \nabla_c Z^{abc}$$

- The black hole entropy is defined as

$$S = \frac{2\pi}{\hbar}\int_{\Sigma} Q^{ab}[\xi] N_{ab} dA$$

Change in entropy and Clausius equation



- If s^{ab} denotes the entropy density associated with a LCH, the change in entropy between two slices of LCH is given as

$$\delta S = -2 \int \nabla_b s^{ab} k_a dV dA.$$

- The Clausius equation is given as

$$-(\hbar/\pi)\nabla_b s^{ab}\, k_a = T^{ab}\xi_b\, k_a + O(x^2).$$

Noetheresque entropy and higher curvature gravity

- Required entropy density for higher curvature equation of state

$$s^{ab} = X^{abcd} \nabla_c \xi_d + W^{abc} \xi_c$$

- Computing the left side of Clausius equation

$$\begin{aligned} \nabla_b s^{ab} &= \left(\nabla_p W^{aps} + X^{apqr} R_{rqp}{}^{s} \right) \xi_s \\ &+ X^{apqr} \left(\nabla_p \nabla_q \xi_r - R_{rqp}{}^{s} \xi_s \right) \\ &+ \left(W^{apq} + \nabla_r X^{arpq} \right) \nabla_p \xi_q . \end{aligned}$$

- This gives the equation of motion(state)

$$-P^{pqr(a} R_{pqr}{}^{b)} + 2_p \nabla_q P^{p(ab)q} + \frac{1}{2} L g^{ab} = -\frac{1}{2} T^{ab}_{(m)} .$$

Modified Noetheresque entropy

- Proposed modification to the Noether charge entropy

$$s^{ab} \;=\; W^{abc}\xi_c + X^{abcd}\nabla_{[c}\xi_{d]} + 2M^{c[a}\xi^{b]}\xi_c.$$

- The additional term can be accounted by using the ambiguity in Noether charge uppon adding a surface term to the Lagrangian. Adding an exact form $d\mu$ where $\boldsymbol{\mu}_{a_1\ldots a_{n-1}} = \boldsymbol{\epsilon}_{a_1\ldots a_{n-1}p} M^{pq}\xi_q$ shifts the Noether charge from Q to $Q + \xi.\mu$
- This term is also the only term that can contribute at $O(x)$ to the change in entropy in the Claussius equation.

Higher derivative gravity

- The equation of motion is given as

$$X^{pqr(a}R_{pqr}{}^{b)} - 2\nabla_p\nabla_q X^{p(ab)q} + \Phi g^{ab} + M^{ab} = -\frac{1}{2}T^{ab},$$

- Entropy must be consistent with Black hole entropy in the proper limit.
- For getting the correct equation of motion for theories containing derivatives of curvature in the action, the choice of M^{ab} will be

$$M^{ab} = \frac{\partial L}{\partial g_{ab}} + 2E^{pqr(a}R_{pqr}{}^{b)} + B^{ab}$$

Non minimally coupled theories

- Using a probe stress tensor for theories where we cannot trivially write "*geometry* $=$ *matter*"
- Suppose there is a term like $R^{ab\nabla_a\phi\nabla_b\phi}$ in the action. There is no natural way of saying if we should take its contribution on the geometry side or entirely on the matter side. This creates an obstruction in defining the heat flux in a consistent way.
- This term will always contribute to the Noether charge and thus will modify the entropy. But then if we also take its contribution on the matter side it doesnt yields the correct equation of motion.
- We avoid this by invoking the concept of a test field probing the geometry and take the entire contribution of this term to the equation of motion through M^{ab}

Discussion

- A new entropy was adapted to the LCH, keeping the black hole entropy unchanged for the theory, so that the Claussius relation is consistent with the equations of motion for the theory.
- Rather than simply invoking the Einstein equivalnce principle, a local inetrial coordimnate system was constructed so that the Killing identity and equation holds upto a order in NNC.
- A test energy momenum was used to probe the dynamics governing the geometry of the horizon and as a result our derivation was further extended to non minimally coupled theories.

THANK YOU