Black Hole Hair: Quantum and Classical

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Skyrmion Black Hole Hair: Conservation of Baryon Number by Black Holes

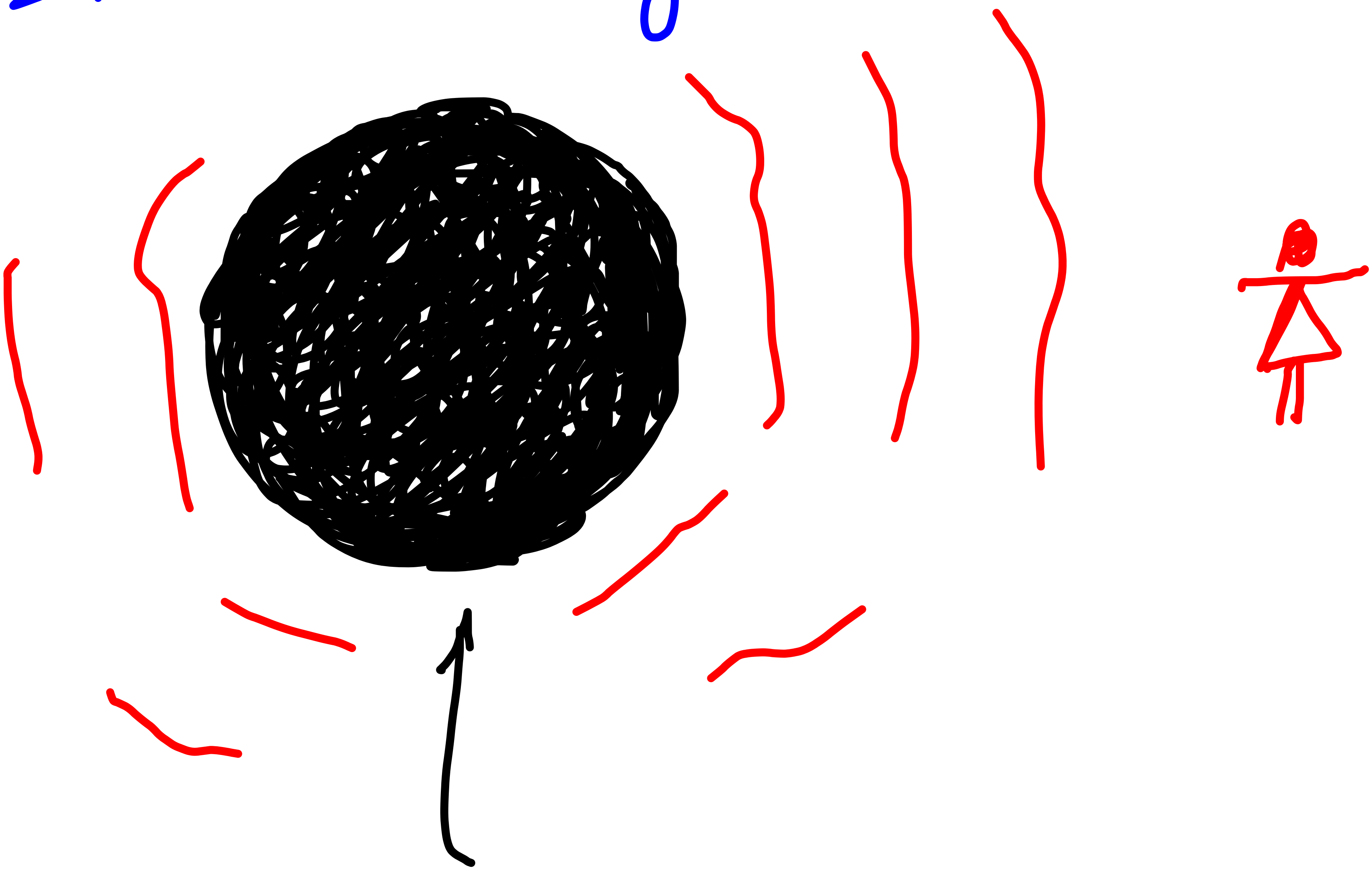
with: Alexander Gußmann

Folk theorem:

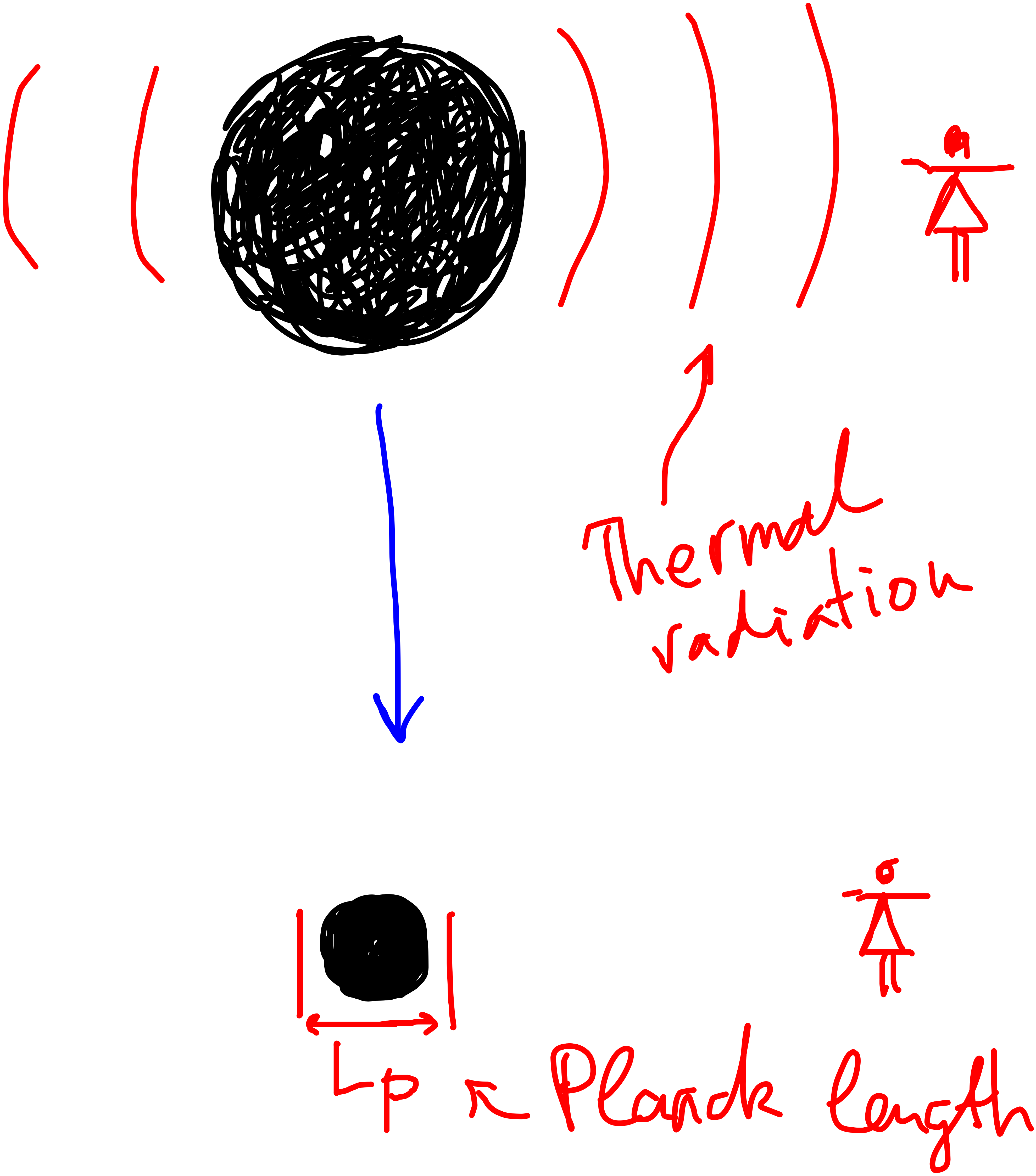
Semi-classical black hole physics is incompatible with conserved baryon number!

We shall find a loophole in the "proof".

Standard argument:



Black hole with baryons in it


Thermal radiation
L_P
Planck length

Assumptions:

- No baryonic hair;

- No deviation from thermality untill

$$x_h \sim L_P$$

Black hole horizon

Skyrmion hair.

$$\mathcal{L}_S = -F_\pi^2 \, Tr\left(U^\dagger \partial_\mu U \, U^\dagger \partial^\mu U\right)$$
$$+ \frac{1}{e^2} Tr\left([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2\right)$$

where

$$U = e^{\frac{i}{F_\pi} \pi^a \cdot \sigma^a} \leftarrow SU(2).$$

$\pi^a \leftarrow$ Pions

Skyrmion size:

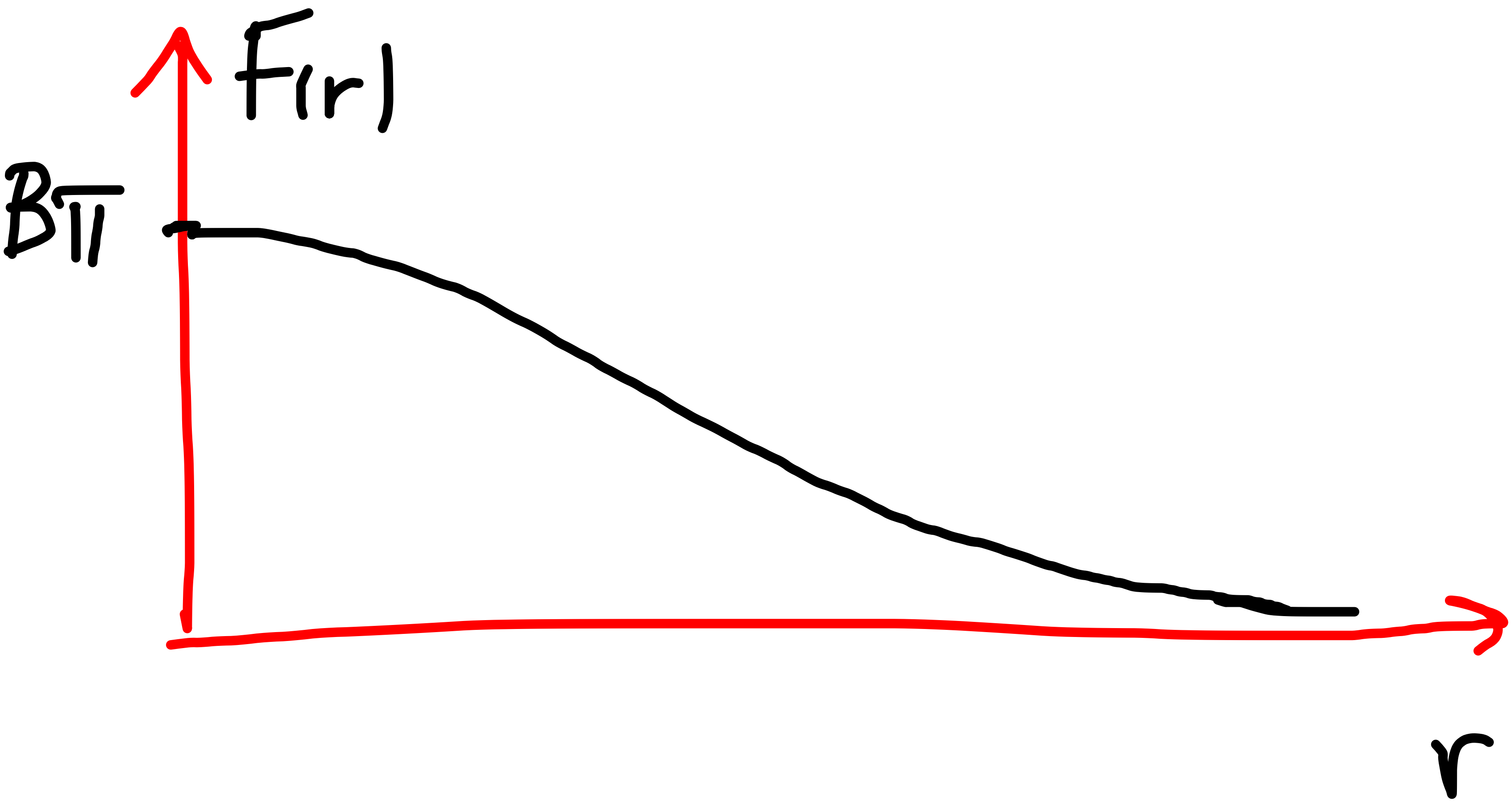
$$L = \frac{1}{e F_\pi}$$

Skyrmion Mass

$$M_s = \frac{F_\pi}{e}$$

Skyrmion:

$$\frac{\pi^a}{F_\pi} = F(r)\, n^a$$



$$F(0) = B\pi, \quad F(\infty) = 0$$

Skyrm topological Chern-Simons current

$$J_\mu \equiv -\varepsilon_{\mu\nu\alpha\beta}\,\mathrm{Tr}\left(U^{-1}\partial^\nu U\, U^{-1}\partial^\alpha U\, U^{-1}\partial^\beta U\right)$$

Topological charge

$$B = \int d\vec{x}\; J_0$$

Black holes with skyrmion hair have been found for

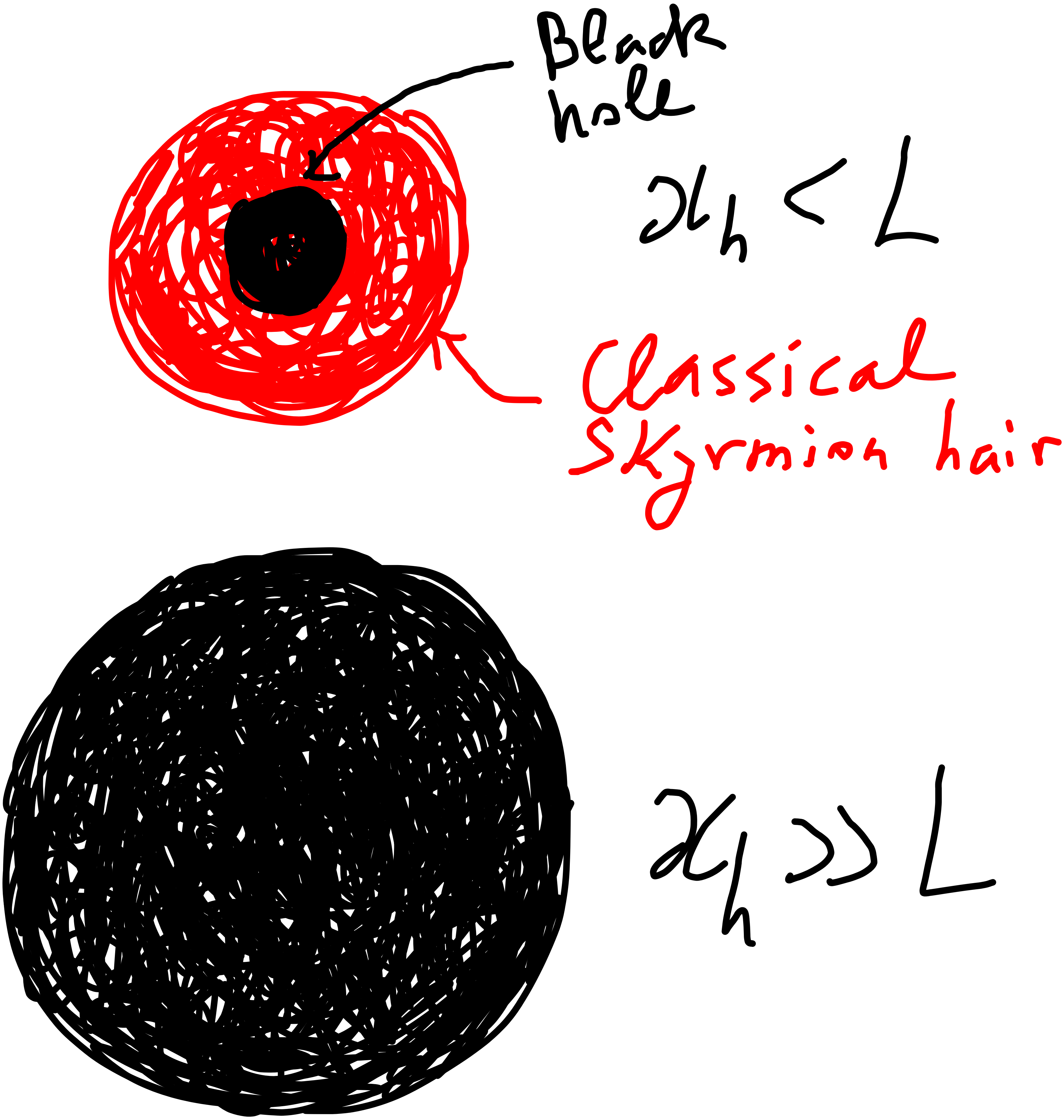
$$x_h \lesssim L$$

Black holes with Skyrmion hair (Luckock, Moss; Droz, Heusler, Straumann; Bizon, Chmaj; Shiiki, Nawado; ...)

$$ds^2 = N^2(r)\left(1 - \frac{2M(r)G_N}{r}\right)dt^2 - \left(1 - \frac{2M(r)G_N}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$$

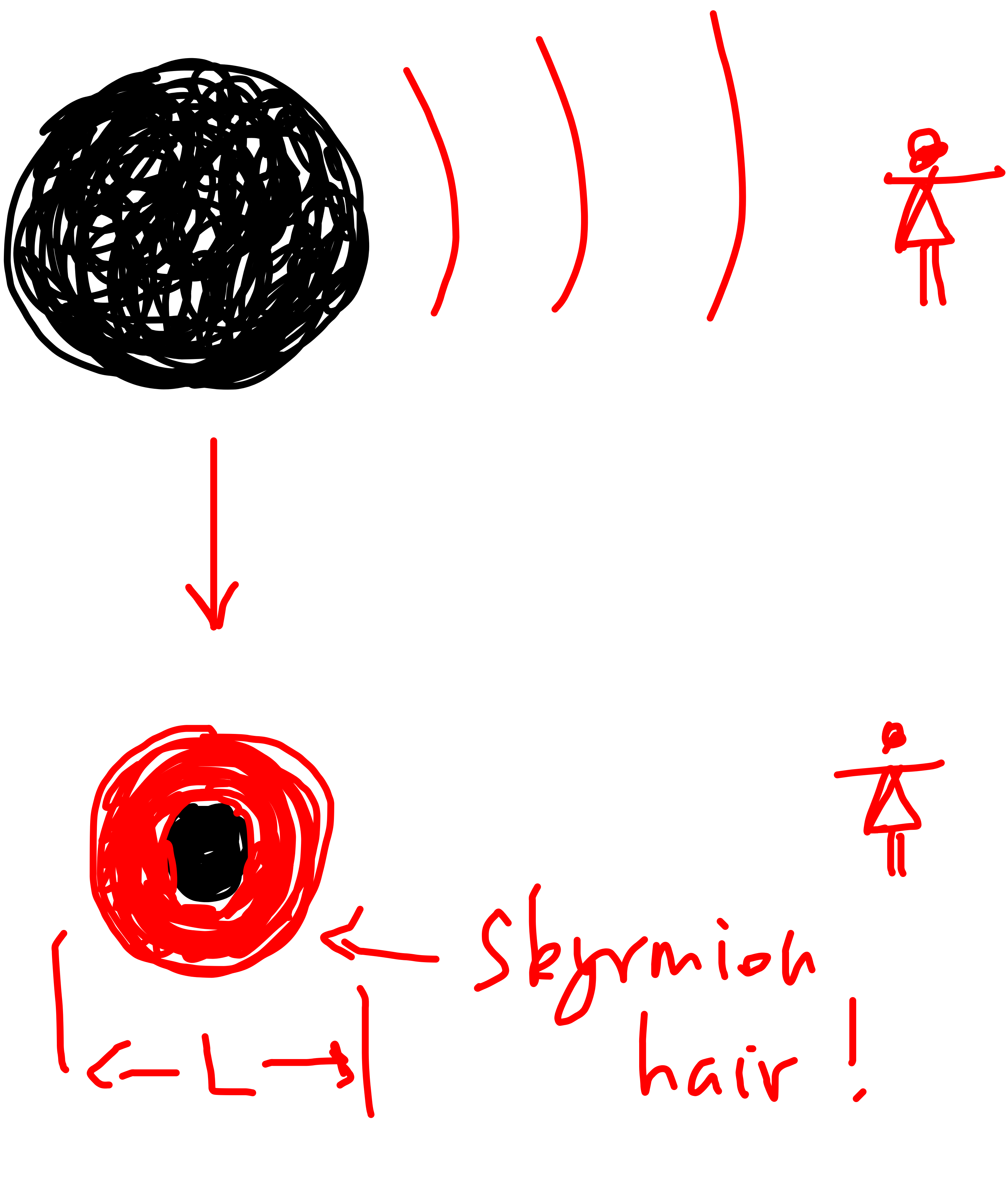
$$N(\infty) = 1 \qquad F(\infty) = 0$$

$$F(r) \propto \frac{1}{r^2} \qquad r \to \infty$$



Black hole
$x_h < L$
Classical Skyrmion hair
$x_h \gg L$

This opens up a logical possibility



Is this only a logical possibility or is a must?

What is the implication for baryon number?

Skyrmion - Baryon correspondence in QCD

$SU(N_c)$. Witten

$U(1)_B \quad q \to e^{i\alpha} q$

Baryon current $J_\mu = \frac{1}{N_c} \bar{q} \gamma_\mu q$

Skyrmion current

$$J_\mu = -\varepsilon_{\mu\nu\alpha\beta} \mathrm{Tr}\left(U^{-1}\partial^\nu U U^{-1}\partial^\alpha U U^{-1}\partial^\beta U\right)$$

Notice

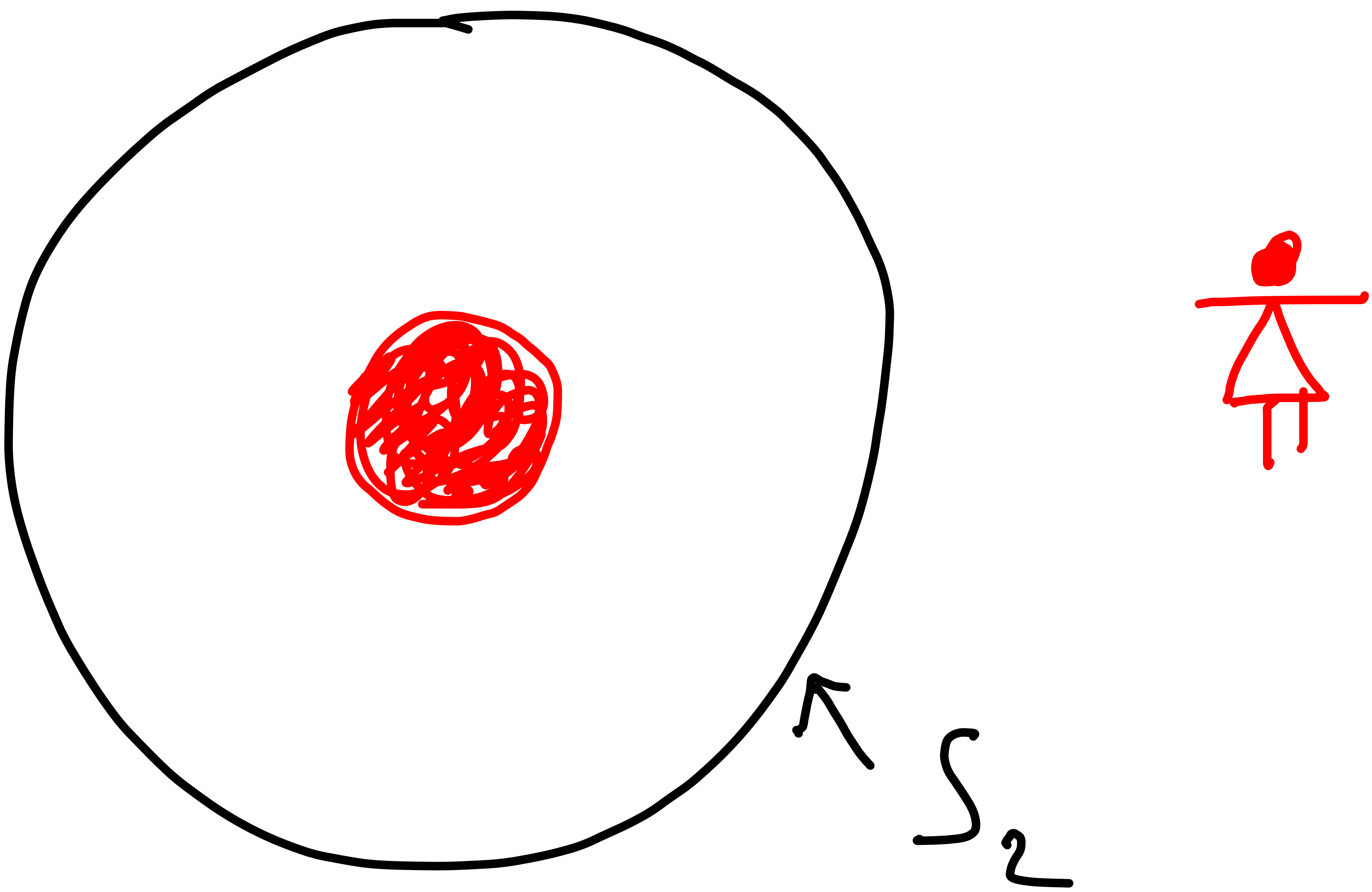
$$J_0 = {}^*d\,S$$

where

$$S_{\mu\nu} = -\left(F(r) - \frac{1}{2}\sin(2F(r)) - \pi\right) \cdot \partial_{[\mu} \cos\theta\, \partial_{\nu]}\phi$$

Thus,

$$B = \int d^3x\, J_0 = \int_{S_2} dx^\mu \wedge dx^\nu\, S_{\mu\nu}$$



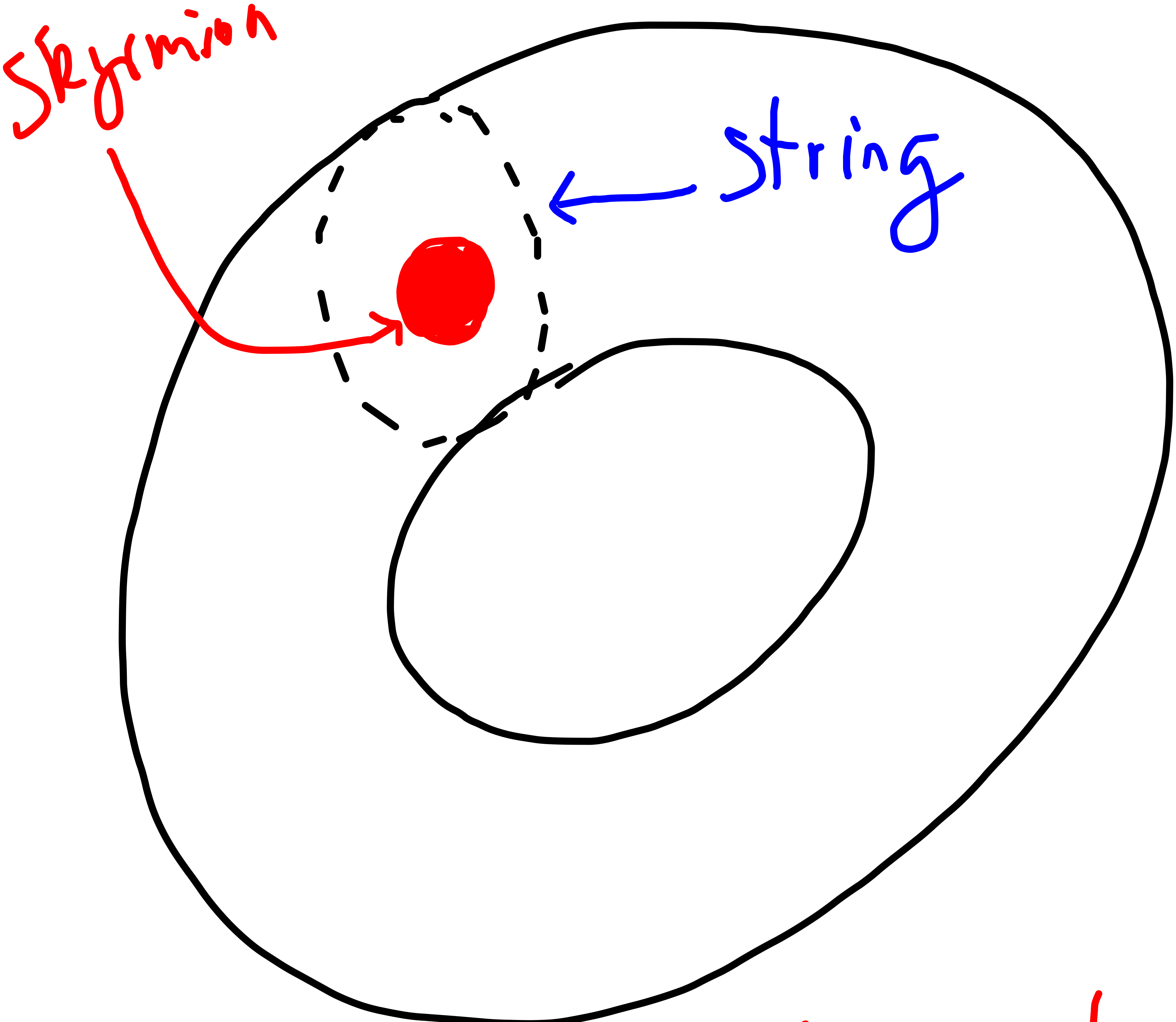
Thus, Skyrmion/baryon charge can be measured by a surface integral over S_2

$$B = \int_{S_2} dx^\mu \wedge dx^\nu \, S_{\mu\nu}$$

For this we couple $\Sigma_{\mu\nu}$ to a probe string

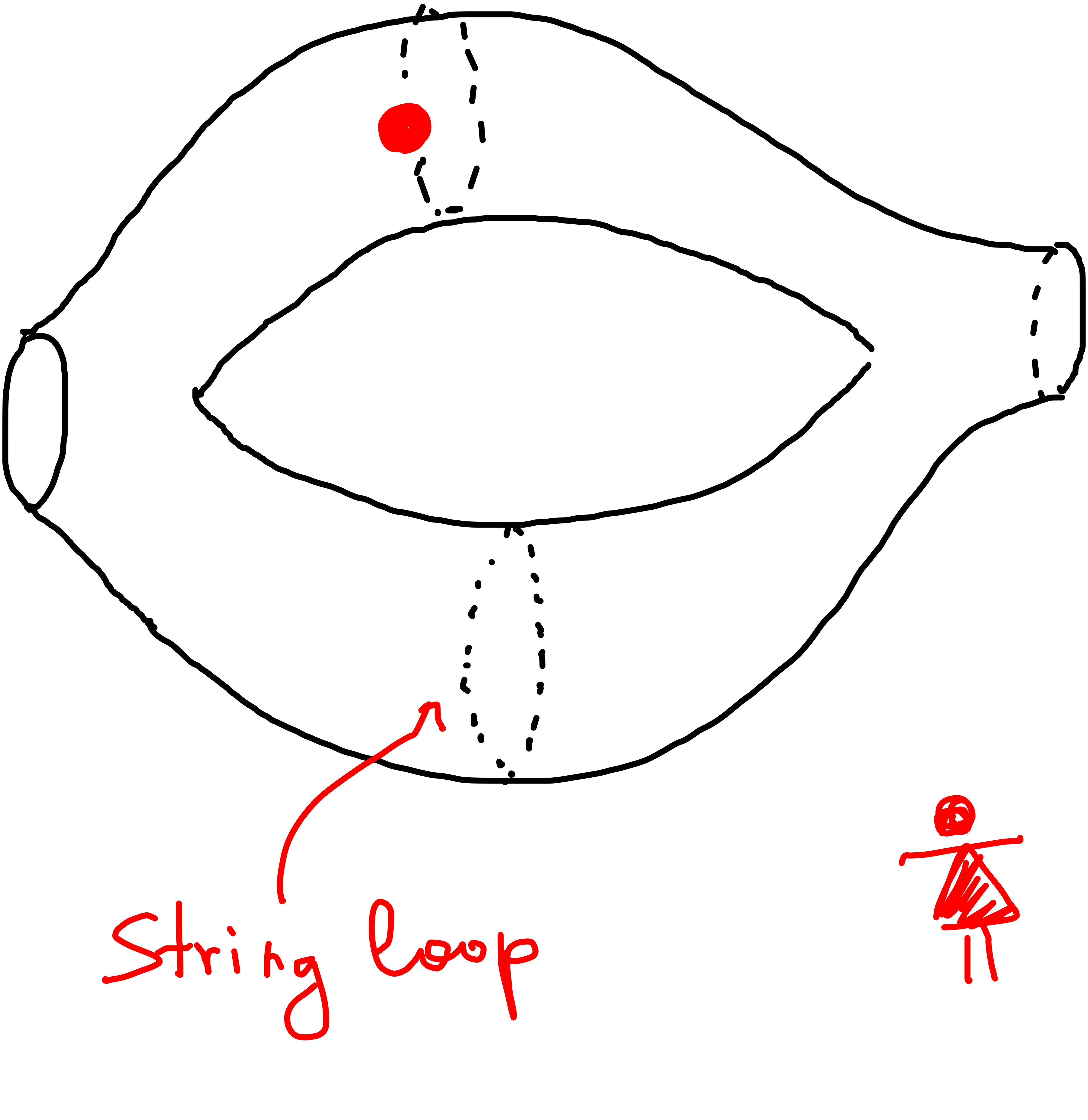
$$S = g \int dX^{\mu} \wedge dX^{\nu} \, \Sigma_{\mu\nu}$$

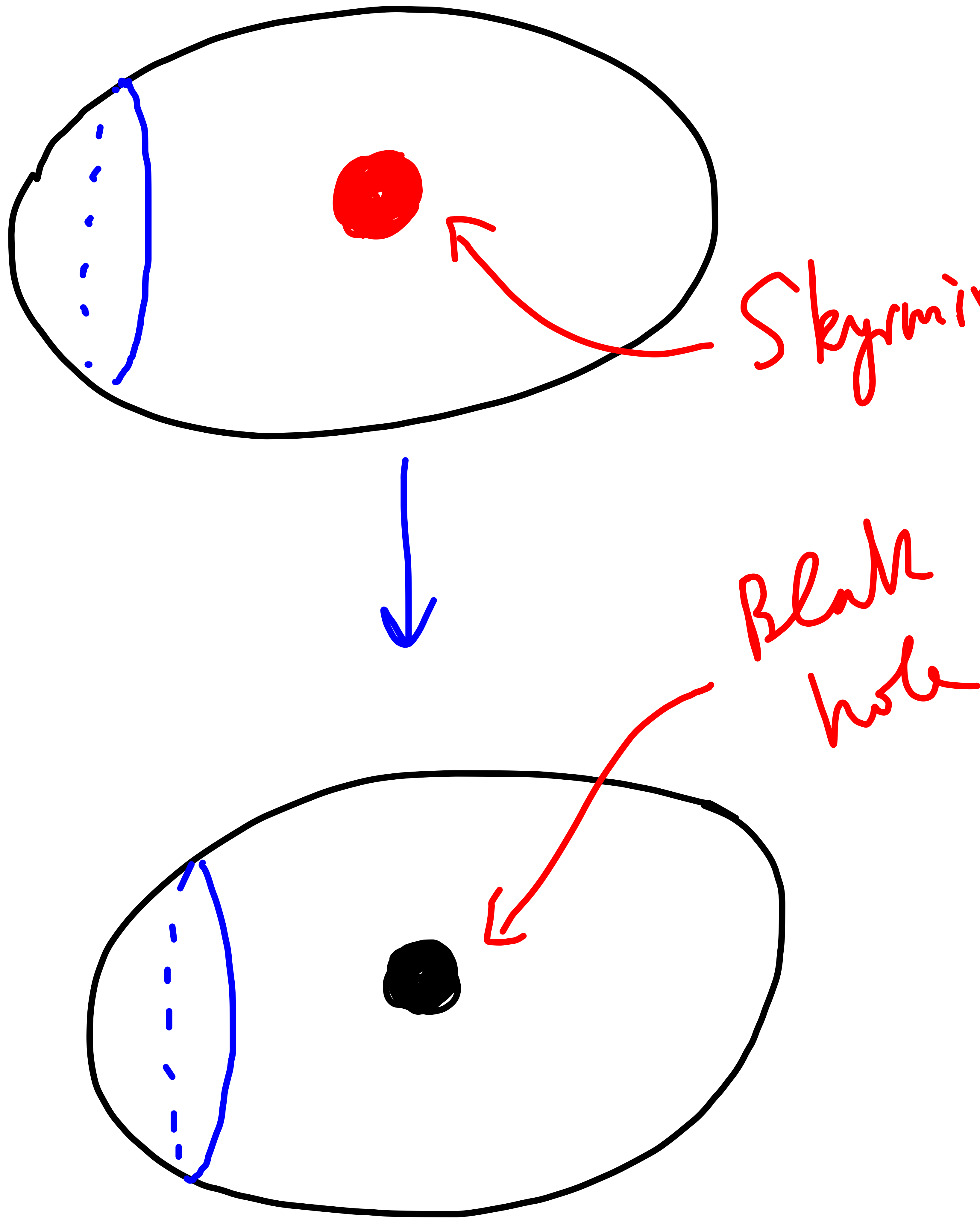
and perform Aharonov-Bohm type interference experiment.



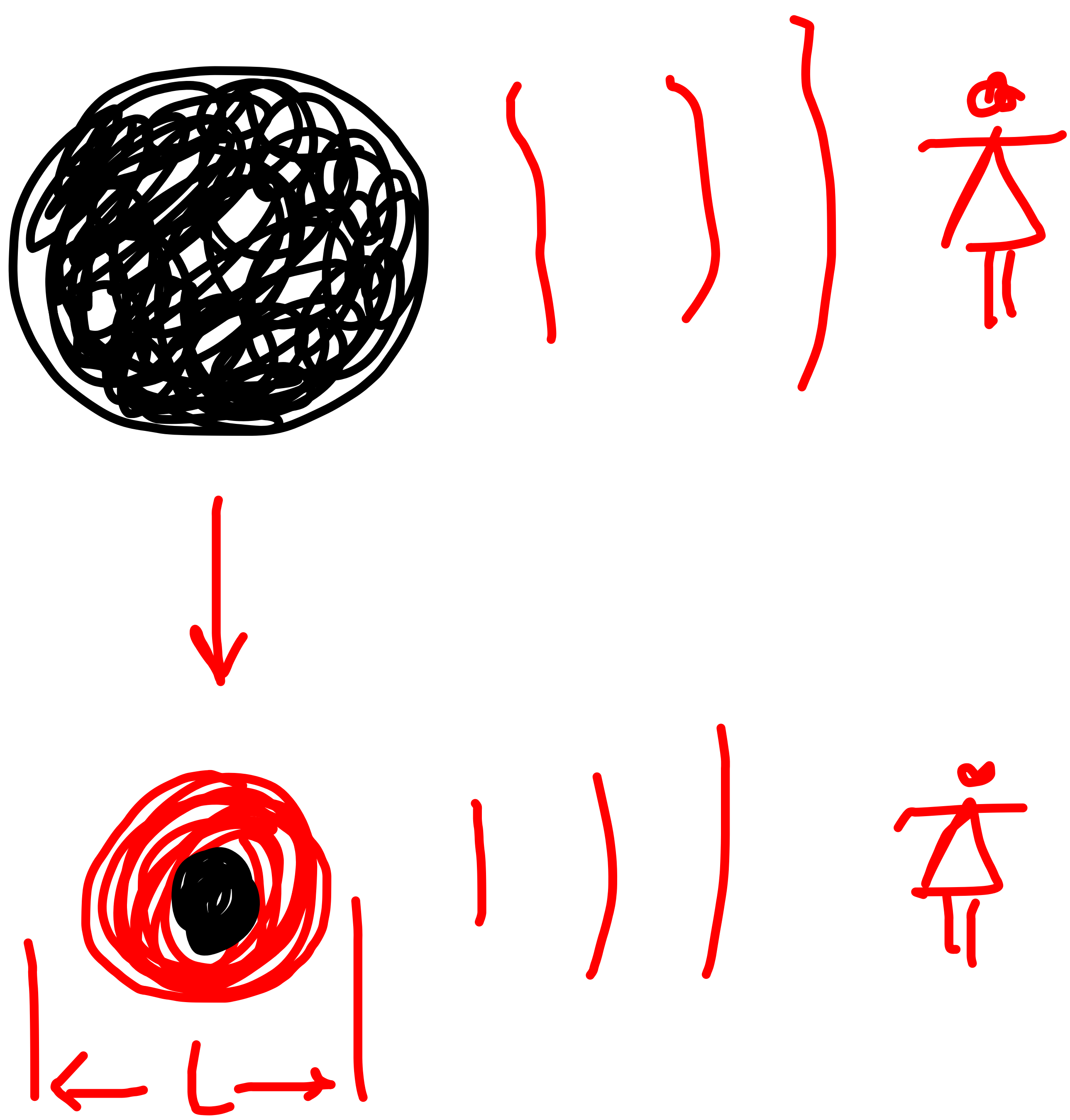
Aharonov-Bohm phase-shift

$$\Delta\Phi = 2\pi g B$$


String loop


Skyrmion
Black hole

Since Alice can measure AB-phse-shift she can monitor Skyrmion/baryon number at $r=\infty$.



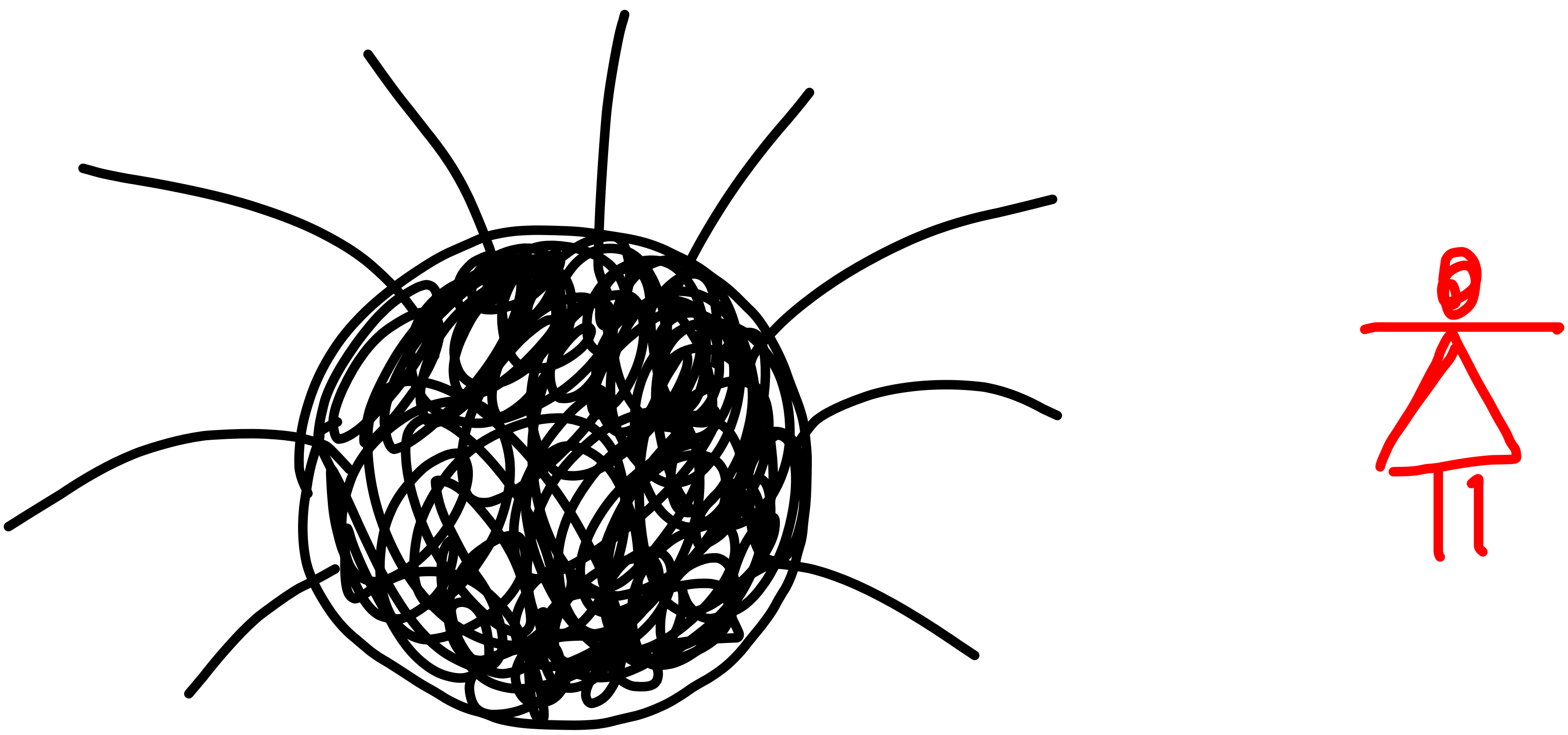
So the baryon number cannot be lost!

Astrophysical implications?

Gravitational Black Hole Hair from Event Horizon Supertranslations.

with:

Averin, Gomez, Lüst



But, in quantum physics black holes carry huge Bekenstein entropy:

$$N = \frac{R^2}{L_p^2}$$

Classical limit $(\hbar \to 0)$

$$N \longrightarrow \infty \, !$$

Howe can be reconcile

$$0 = \infty \quad ?$$

Our approach:

The degrees of freedom that store information must decouple as

$$\frac{1}{N}, \text{ for } N \to \infty .$$

Then in classical limit ($\hbar \to 0$) black hole indeed carries an infinite entropy and information.

But, it takes $t = \infty$ to decode this info!

Who are these information-carrier degrees of freedom?

Today: we will consider purely gravitational hair, as it is relevant for the Schwarzschild black hole.

Slightly different definition of hair:

(Almost) degeneracy of the black hole metrics (vacua).

Infinite classical hair: non-uniqueness of the black hole metric:

New kind of event horizon supertranslations provide an infinite family of black holes metrics:

$\mathcal{A}$ - supertranslations: $\delta_{\mathcal{A}}\ g_{\mu\nu}^{BH} = \tilde{g}_{\mu\nu}^{BH}$

The $\mathcal{A}$ - supertranslation group is spontaneously broken.

Infinitely many massless Goldstone modes

$\Rightarrow$ infinite classical black hole entropy:

$S_{class} = \infty$ $\qquad\qquad$ $Q_{class}^{\mathcal{A}} = 0$

Finite quantum hair: non-uniqueness of finitely many almost degenerate black hole vacua:

$\mathcal{A}$ - supertranslations generators generate transitions among the black hole vacua:

$$\mathcal{T}^{\mathcal{A}} \, |BH\rangle = |\widetilde{BH}\rangle$$

Finitely many charges which correspond to finitely many pseudo Goldstone modes.

$Q^{\mathcal{A}}_{q.m.} \neq 0, \quad \mathcal{S}_{q.m.} = N \quad \Rightarrow$ finite quantum hair.

II) Gravitational Black Hole Hair from Event Horizon Supertranslations

[G. Dvali, C. Gomez, D.L. arXiv.1509.02114;
A. Averin, G. Dvali, C. Gomez, D.L., arXiv:1601.03725;
related work: S. Hawking, arXiv:1509.01147;
S. Hawking, M. Perry, A. Strominger, arXiv:1601.00921]

Some basic definitions:

Schwarzschild radius: $r_S \equiv 2G_N M$

Planck length: $L_P^2 \equiv \hbar G_N$

Entropy: $\mathcal{S} \sim N = \frac{r_S^2}{L_P^2}$

Classical limit:

$$\hbar \to 0\,, M = \text{finite}, \;\; r_S = \text{finite}$$

Semiclassical limit:

$$\hbar = \text{finite}, \;\; r_S = \text{finite}, \;\; G_N \to 0, \;\; M \to \infty$$

$\Big\} \; N \to \infty$

Note that Minkowski space can be regarded as the near horizon limit of the Schwarzschild geometry, obtained in the limit $r_S \longrightarrow \infty$.

In quantum language it means that the corresponding Minkowski vacua are infinitely degenerate and can be regarded as coherent state of infinitely many gravitons with zero momentum:

[G. Dvali, C. Gomez, D.L., arXiv:1509.02114]

$$\mathcal{T}^{\mathcal{BMS}^-} |Min\rangle = |\widetilde{Min}\rangle$$

Schwarzschild metric in (infalling) Edington - Finkelstein coordinates:

$$ds^2 = -(1 - \frac{r_S}{r})dv^2 + 2dvdr + r^2 d\Omega^2$$

$$v = t + r^* \qquad dr^* = (1 - \frac{r_S}{r})^{-1} dr$$

Supertranslations:

Diffeomorphisms that correspond to vector fields

$$\zeta^\mu = (f, A, B, C)$$

with $\delta_f g_{\mu\nu} = \tilde{g}_{\mu\nu}$

(iii) $\mathcal{A}$ - supertranslations: microstates of black hole:

Compare supertranslations on $\mathcal{I}^-$ and on $\mathcal{H}$:

Horizon supertranslations contain a part which cannot be compensated by standard BMS-supertranslations.

Belong to the factor space: $\mathcal{A} \equiv BMS^{\mathcal{H}}/BMS^-$:

$$
\begin{aligned}
\chi_f^\mu &= \zeta_f^\mu - \eta_f^\mu \\
&= (0, 0, -\frac{1}{r_S}\frac{\partial f}{\partial \vartheta}, -\frac{1}{r_S \sin^2\vartheta}\frac{\partial f}{\partial \varphi})
\end{aligned}
$$

These transformations are intrinsically due to the presence of the horizon.

The corresponding fluctuations of the metric correspond to physical massless modes of the black hole:

$$\delta_{\chi_f} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2r^2 \frac{1}{r_S} \frac{\partial^2 f}{\partial \vartheta^2} & -2r^2 \frac{1}{r_S} (\frac{\partial^2 f}{\partial \vartheta \partial \varphi} - \cot \vartheta \frac{\partial f}{\partial \varphi}) \\ 0 & 0 & * & -2r^2 \frac{1}{r_S} (\frac{\partial^2 f}{\partial \varphi^2} + \sin\theta \cos\vartheta \frac{\partial f}{\partial \vartheta}) \end{pmatrix}$$

We call them gapless Bogoliubov $\mathcal{A}$ - modes.

They correspond to gravitational waves along the horizon with infinite wave length.

$\Rightarrow$ Massless Goldstone modes of $\mathcal{A}$ - supertranslations.

- They keep the ADM mass of the black hole invariant .
- The are solutions of the full non-linear Einstein equations.

(A. Gußmann)

Now we return to the black hole:

The $\mathcal{A}$ - supertranslation generators transform one black hole vacuum into another one:

$$\mathcal{T}^{\mathcal{A}}\,|BH\rangle = |\widetilde{BH}\rangle$$

Finite number of quantum hair: $\mathcal{S}$, N become finite !

Finite number of non-zero charges $\mathcal{Q}^{\mathcal{A}}_{q.m.}$

Charges $\mathcal{Q}^{\mathcal{A}}_{q.m.}$ measure the capacity to resolve the hair.

How does the quantum hair become finite?

How many angular Bogoliubov modes must be counted as information carriers?

Need microscopic quantum picture for black hole:

Black hole: bound state of N soft gravitons at a quantum critical point.

[G. Dvali, C. Gomez, 2011]

- Finite energy gap between the different black hole vacua.

$$\Rightarrow$$

$\mathcal{A}$ - supertranslations are explicitly broken.

Goldstone modes acquire a mass.

Bogoliubov excitations = pseudo Goldstone bosons.

Construction of $\mathcal{A}$ - charges and black hole vacua:

In quantum theory the $\mathcal{A}$ - modes correspond to the following operator valued functions:

$$\delta_{\chi_f}\hat{g}_{\mu\nu}(\vartheta,\varphi) = \sum_{l,m}\left(\hat{b}_{lm}Y_{lm}(\vartheta,\varphi)e^{-iv\omega_{lm}} + \hat{b}^{\dagger}_{lm}Y^{*}_{lm}(\vartheta,\varphi)e^{iv\omega_{lm}}\right)$$

$$\hat{\mathcal{Q}}^{\mathcal{A}}_{lm} \sim \partial_v\left(\delta_{\chi_f}\hat{g}_{\mu\nu}(v,\vartheta,\varphi)\right) = -i\sqrt{\hbar\omega_{lm}}\left(e^{-i\omega_{lm}v}\hat{b}_{lm} - e^{i\omega_{lm}v}\hat{b}^{\dagger}_{lm}\right)$$

Black hole vacua: coherent state of Bogoliubov modes:

$$|BH\rangle \equiv |N\rangle \sim e^{-\sum_{lm}\sqrt{n_{lm}}(\hat{b}_{lm}-\hat{b}^{\dagger}_{lm})}|0\rangle$$

Now it follows that

$$Q^A_{lm} = \langle N|\hat{Q}^A_{lm}|N\rangle \sim \sqrt{\hbar\omega_{lm}n_{lm}} \sim \frac{1}{N}$$

For finite N, the charges are non-vanishing, since

$$\omega_{lm} \sim \Delta E \leq \frac{1}{N}$$

How many charges are there?

The energy gap can be estimated as $\Delta E \sim \frac{l^2}{N^2}\frac{\hbar}{r_S}$.

$$\Rightarrow \quad l_{max} \sim \sqrt{N}$$

$-l \leq m \leq l \quad \Rightarrow$ There exist $l^2 = N$ different charges.

These charges should be preserved during the Hawking radiation.

What is the corresponding entropy?

There exist $l^2 = N$ different Boguliubov modes b_{lm} .

Each Bogoiubov qubit carries (at least) one bit of information.

$\Rightarrow$ The number of states is 2^N .

$\Rightarrow$ Entropy $S \sim N$

This is in agreement with the Bekenstein Hawking entropy.

Immediate consequences of this portrait:

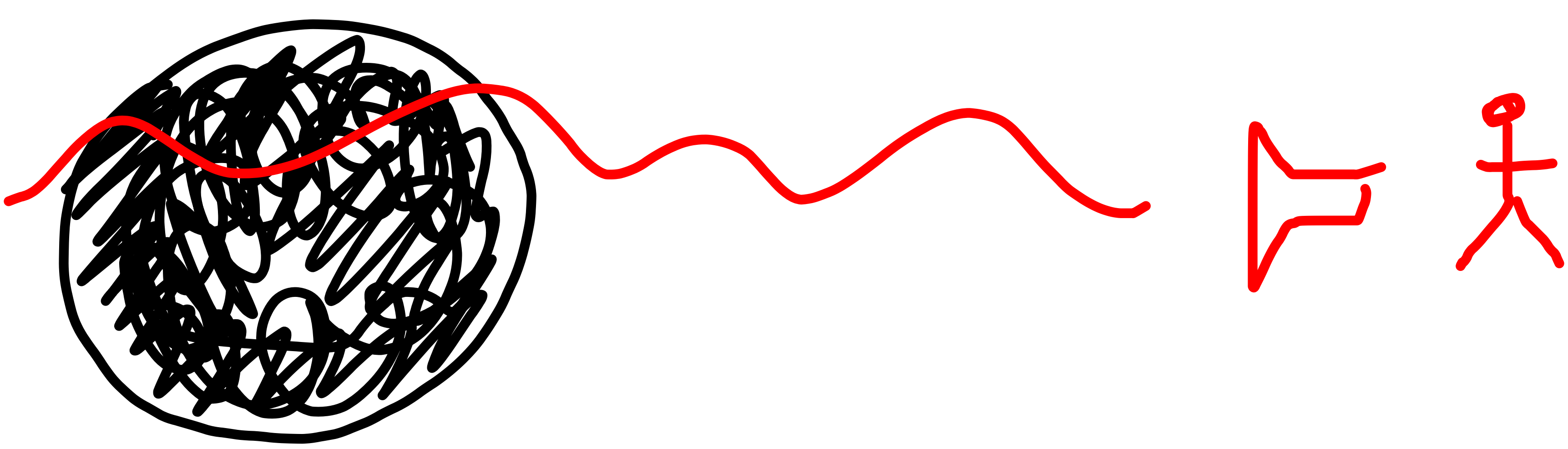
$\circledast$ Black holes carry hair!

The amount of hair is <u>infinite</u> in classical limit:

$N \to \infty$

But, in classical limit ($N \to \infty$) the time-scale required for resolving the hair is also infinite!

$$t \sim N^2 \longrightarrow \infty$$



Solutions:

$$BMS^{\mathcal{H}} \ : \quad \zeta_f^\mu = \Big(f(\vartheta,\varphi), 0, \frac{\partial f}{\partial \vartheta}(\frac{1}{r} - \frac{1}{r_S}), \frac{1}{\sin^2\vartheta}\frac{\partial f}{\partial \varphi}(\frac{1}{r} - \frac{1}{r_S}) \Big)$$

Variation of metric:

$$\delta_{\zeta_f} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & -(1-\frac{r_S}{r})\frac{\partial f}{\partial\vartheta} & -(1-\frac{r_S}{r})\frac{\partial f}{\partial\varphi} \\ 0 & 0 & 0 & 0 \\ * & * & 2r^2(\frac{1}{r}-\frac{1}{r_S})\frac{\partial^2 f}{\partial\vartheta^2} & 2r^2(\frac{1}{r}-\frac{1}{r_S})(\frac{\partial^2 f}{\partial\vartheta\partial\varphi} - \cot\vartheta\frac{\partial f}{\partial\varphi}) \\ * & * & * & 2r^2(\frac{1}{r}-\frac{1}{r_S})(\frac{\partial^2 f}{\partial\varphi^2} + \sin\vartheta\cos\vartheta\frac{\partial f}{\partial\vartheta}) \end{pmatrix}$$

(i) Supertranslations at the horizon $\mathcal{H}$:

Boundary conditions:

[see also: S. Hawking, arXiv:1509.01147; M. Perry, talk 2015;
S. Hawking, M. Perry, A. Strominger, arXiv:1601.00921;
L. Donnay, G. Giribet, H. Gonzales, M. Pino, arXiv:1511.08687
M. Blau, M. O‘ Loughlin, arXiv:1512.02858]

The supertranslations should preserve the structure of the metric at the horizon:

$$f|_{r=r_S} = f(\theta, \phi)\,,$$

$$A|_{r=r_S} = B|_{r=r_S} = C|_{r=r_S} = 0$$

Furthermore we require that $\delta_f g_{r\mu} = 0$

$\Rightarrow$ DGL‘s

$$\begin{aligned}
\frac{\partial f}{\partial r} &= 0\,,\\
\frac{\partial A}{\partial r} &= 0\,,\\
\frac{\partial}{\partial r}(r^2 B) + \frac{\partial f}{\partial \vartheta} - 2rB &= 0\,,\\
\frac{\partial}{\partial r}(r^2 \sin^2(\vartheta) C) + \frac{\partial f}{\partial \varphi} - 2r\sin^2(\vartheta)C &= 0.
\end{aligned}$$

(ii) Standard BMS supertranslations at $\mathcal{I}^-$:

Diffeomorphisms that preserve the structure of the metric at past null infinity.

There are usually considered in Bondi coordinates.

For the Schwarzschild metric they are generated by the following vector fields:

$$BMS^- : \quad \eta_g^\mu = \left(g(\vartheta, \varphi), 0, \frac{\partial g}{\partial \vartheta} \frac{1}{r}, \frac{1}{\sin^2 \vartheta} \frac{\partial g}{\partial \varphi} \frac{1}{r} \right)$$

Variation of black hole metric:

$$\delta_{\eta_g} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & -(1-\frac{r_S}{r})\frac{\partial g}{\partial \vartheta} & -(1-\frac{r_S}{r})\frac{\partial g}{\partial \varphi} \\ 0 & 0 & 0 & 0 \\ * & * & 2r\frac{\partial^2 g}{\partial \vartheta^2} & 2r(\frac{\partial^2 g}{\partial \theta \partial \varphi} - \cot\vartheta \frac{\partial g}{\partial \varphi}) \\ * & * & * & 2r(\frac{\partial^2 g}{\partial \varphi^2} + \sin\theta\cos\vartheta\frac{\partial g}{\partial \vartheta}) \end{pmatrix} .$$

They even act non-trivially on Minkowski space-time:

Infinite family of flat Minkowski metrics.