Lovelock Thermodynamics (and Holography)

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H-QM Helmholtz Research School
Quark Matter Studies

FIAS Frankfurt Institute for Advanced Studies

Lovelock gravity

Gravitational theories with higher derivative terms:

- E.o.m. contain more than two derivatives of the metric
- Hard to solve exactly
- Additional degrees of freedom

There exists a special class of gravitational theories with higher derivative terms, Lovelock gravity

- E.o.m. contain only up to second order derivatives of the metric.
- Holographic point of view:

  The higher curvature terms correspond, on the gauge theory side of the AdS/CFT, to corrections due to finite N (rank of the gauge group) and finite t'Hooft coupling.

Lovelock gravity

Extension of Einstein theory to higher dimensions

- Great interest in theoretical physics as it describes a wide class of models
- Admits the Einstein general relativity and the so called Chern-Simons theories of gravity as particular cases

- Einstein-Gauss-Bonnet gravity = 2nd-order Lovelock
- 5d GB gravity can be used to describe 1/N corrections to relativistic 4d QFTs with a gravitational dual.
- In the hydrodynamic limit the theory describes a “GB plasma” and transport coefficients can be calculated using AdS/CFT

Maxwell- Lovelock gravity

$$I = \frac{1}{16\pi G_N}\int d^dx\sqrt{-g}\Big(\sum_{k=0}^{k_{max}} \hat{\alpha}_{(k)}\mathcal{L}^{(k)} - 4\pi G_N F_{ab}F^{ab}\Big)$$

Lagrangian of a Lovelock gravity in d spacetime dim

$$\mathcal{L} = \frac{1}{16\pi G}\sum_{k=0}^{k_{max}} \hat{\alpha}_{(k)}\mathcal{L}^{(k)}$$

$$k_{max} = \left[\frac{d-1}{2}\right]$$

$$\mathcal{L}^{(k)} = 0 \quad d < 2k$$

Given by the contraction of k powers of the Riemann tensor

$$\mathcal{L}^{(k)} = \frac{1}{2^k}\delta^{a_1 b_1 \ldots a_k b_k}_{c_1 d_1 \ldots c_1 d_1} R_{a_1 b_1}{}^{c_1 d_1} \ldots R_{a_k b_k}{}^{c_k d_k}$$

- The term k=0 ⟹ Cosmological Constant $\mathcal{L}_0 = -2\Lambda$
- The term k=1 ⟹ Einstein-Hilbert action $\mathcal{L}_1 = R$

Gauss – Bonnet and Third order Lovelock gravity

2[nd] order Lovelock term (Gauss- Bonnet)

$$\mathcal{L}_2 = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$$

contributes to e.o.m. for $d \geq 5$

3[rd] order Lovelock term

$$\begin{aligned}\mathcal{L}_3 = &\ 2R^{\rho\sigma\kappa\lambda}R_{\kappa\lambda\mu\nu}R^{\mu\nu}{}_{\rho\sigma} + 8R^{\rho\sigma}{}_{\kappa\mu}R^{\kappa\lambda}{}_{\sigma\nu}R^{\mu\nu}_{\rho\lambda} + \\ + &\ 24R^{\rho\sigma\kappa\lambda}R_{\kappa\lambda\sigma\mu}R^{\mu}_{\rho} + 3RR^2_{\rho\sigma\kappa\lambda} + 24R^{\rho\kappa\sigma\lambda}R_{\sigma\rho}R_{\lambda\kappa} + \\ + &\ 16R^{\rho\sigma}R_{\sigma\kappa}R^{\kappa}_{\rho} - 12RR^2_{\rho\sigma} + R^3\end{aligned}$$

Rescaled Lovelock coupling constant:

$$\alpha_0 = \frac{\hat{\alpha}_{(0)}}{(d-1)(d-2)}, \quad \alpha_1 = \hat{\alpha}_{(1)}, \quad \alpha_k = \hat{\alpha}_{(k)} \prod_{n=3}^{2k} (d-n) \quad \text{for} \quad k \geq 2$$

Charged Lovelock Black Holes

The static charged spherically symmetric AdS Lovelock BHs

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2 d\Omega^2_{(\kappa)d-2} \qquad F = \frac{Q}{r^{d-2}} dt \wedge dr$$

BH horizons geometries ← $\kappa = \pm 1, 0$

Constant Curvature ← $(d-2)(d-3)\kappa$

$$\sum_{k=0}^{k_{max}} \hat{\alpha}_{(k)} \mathcal{G}^{(k)}_{ab} = 8\pi G_N \left(F_{ac} F_b{}^c - \frac{1}{4} g_{ab} F_{cd} F^{cd} \right)$$

The field eqs reduce to the requirement that the metric function solves the polynomial equation of degree k_{max}

$$\mathcal{P}(f) = \sum_{k=0}^{k_{max}} \alpha_k \left(\frac{\kappa - f}{r^2} \right)^k = \frac{16\pi G_N M}{(d-2)\Sigma^{(\kappa)}_{d-2} r^{d-1}} - \frac{8\pi G_N Q^2}{(d-2)(d-3)} \frac{1}{r^{2d-4}}$$

Thermodynamical quantities in Lovelock gravity

Using the **Hamiltonian formalism** it is possible to derive

- The expression for gravitational entropy in Lovelock gravity and the corresponding *first law of black hole thermodynamics*

$$M = \frac{\Sigma_{d-2}^{(\kappa)}(d-2)}{16\pi G_N}\sum_{k=0}^{k_{max}} \alpha_k \kappa^k r_+^{d-1-2k} + \frac{\Sigma_{d-2}^{(\kappa)}}{2(d-3)}\frac{Q^2}{r_+^{d-3}}$$

$$T = \frac{1}{4\pi r_+ D(r_+)}\left[\sum_k \kappa\alpha_k(d-2k-1)\left(\frac{\kappa}{r_+^2}\right)^{k-1} - \frac{8\pi G_N Q^2}{(d-2)r_+^{2(d-3)}}\right]$$

$$S = \frac{\Sigma_{d-2}^{(\kappa)}(d-2)}{4G_N}\sum_{k=0}^{k_{max}}\frac{k\kappa^{k-1}\alpha_k r_+^{d-2k}}{d-2k}$$

5d Maxwell- GB Black Brane (BB) Thermodynamics

Charged GB BB thermodynamics **indistinguishable** RN BB thermodynamics

If expressed in terms of effective physical parameters

$$ds^2 = -f(r)\,N_\sharp^2 dt^2 + f(r)^{-1}\,dr^2 + \frac{r^2}{L^2}d\Sigma_{d-2}^2$$

Rescaling of the time coordinate $t \rightarrow N_\sharp t$ necessary to have a unit c in the dual CFT

$$M = \frac{M_{\rm ADM}}{N_\sharp}, \qquad T = \frac{T_H}{N_\sharp}$$

N is fixed in terms of $\alpha_{k\geq 2}$

⇒ The thermodynamic expressions do not depend on the Lovelock $\alpha_k\,,\, k\geq 2$ (only for $\kappa = 0$)

5d GB Black Brane (BB)
Near – horizon limit

- The extremal, near-horizon limit of the two classes of branes is the same:

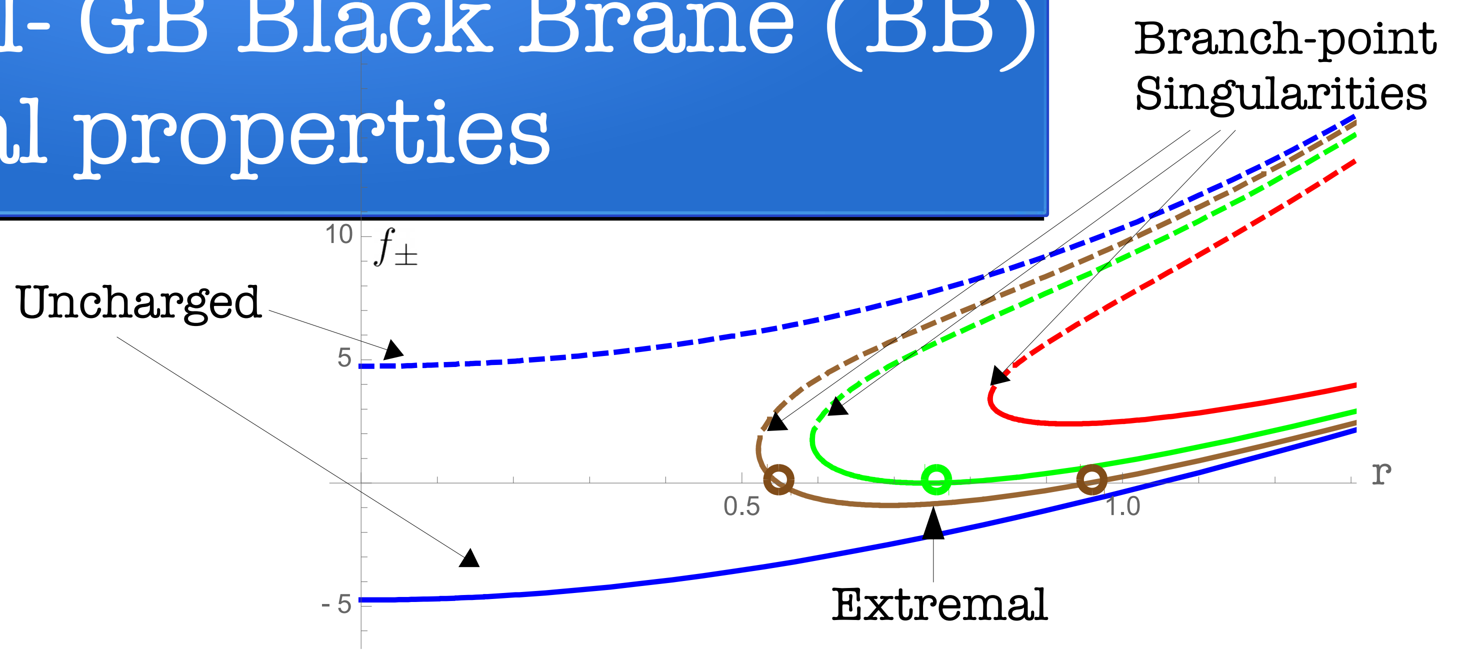
  they allow ($AdS_2 \times R_3$), near-horizon, exact solution

- In the near horizon regime the contributions of the higher-curvature terms to the field equations vanish

- Although in the UV the associated dual QFTs are different, they flow in the IR to the same fixed point!

5d Maxwell- GB Black Brane (BB) Geometrical properties

$\alpha_2 > 0$



Two branches

$$\alpha_2 \frac{(\kappa - f)^2}{r^4} + \frac{(\kappa - f)}{r^2} + \alpha_0 - \frac{\omega_d M}{r^{d-1}} + \frac{8\pi G_N Q^2}{(d-2)(d-3)r^{2d-4}} = 0$$

The theory allows for two branches of solutions continuously connected trough a branch-point singularity

Holographically they represent the flows between two different CFTs through a singularity

GB plasma η/s

- In the non-extremal case we find a non-universal, monotonically increasing ($\alpha_2 < 0$)/ decreasing ($\alpha_2 > 0$) temperature dependent
- Temperature dependent expression for η/s

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - 4\lambda \frac{\pi L^2}{N r_+(T_H, Q)} T_H\right]$$

- In the extremal case we find the universal value

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

2nd-order Lovelock

Gauss – Bonnet

Thermodynamics of topological black holes in R^2 gravity
G. Cognola, M. Rinaldi, L. Vanzo, S. Zerbini

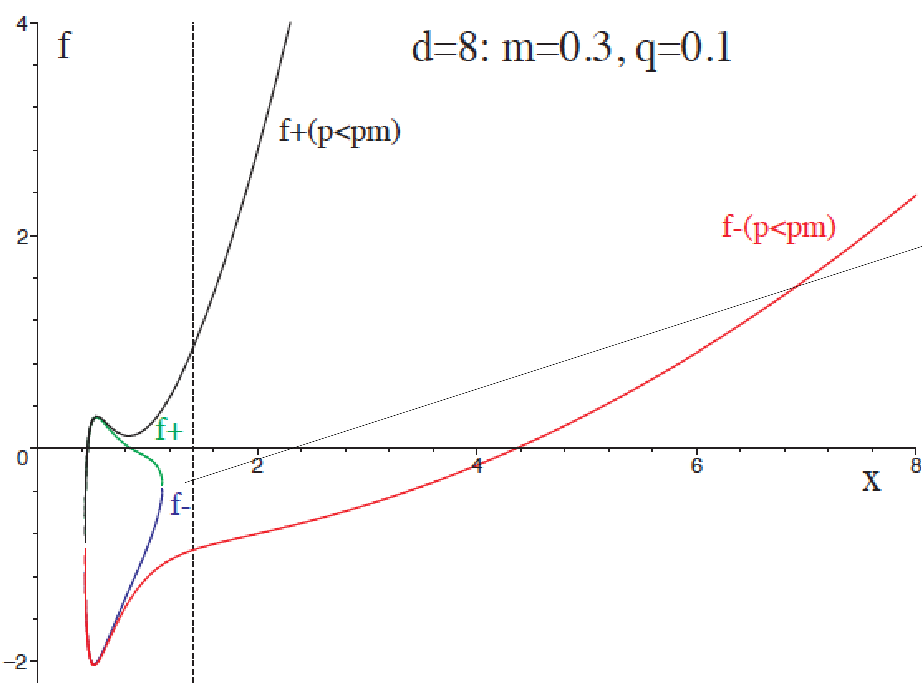
Only for sufficiently small pressure the metric possesses an asymptotic AdS region: $p \leq p_{\max}$

The metric function is the solution of this polynomial:

$$x^{d-5}(\kappa - f)^2 + x^{d-3}(\kappa - f) + \alpha_2\alpha_0 x^{d-1} - m + \frac{4\pi q^2}{(d-2)(d-3)x^{d-3}} = 0$$

$\kappa = -1$

(Shift upward for +1)



The nonlinear curvature is too strong and the space becomes compact.

- No asymptotic AdS region,
- No `proper' BH with `standard' asymptotics.

Black holes as thermodynamic objects

$$\text{Energy} \quad E \quad \leftrightarrow \quad M \quad \text{Mass}$$

$$\text{Temperature} \quad T \quad \leftrightarrow \quad \frac{\kappa}{2\pi} \quad \text{Surface Gravity}$$

$$\text{Entropy} \quad S \quad \leftrightarrow \quad \frac{A}{4} \quad \text{Horizon Area}$$

$$dE = TdS \boxed{- PdV} + \text{work terms} \quad \leftrightarrow \quad dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ$$

Mass of a black hole is interpreted as the enthalpy of spacetime.

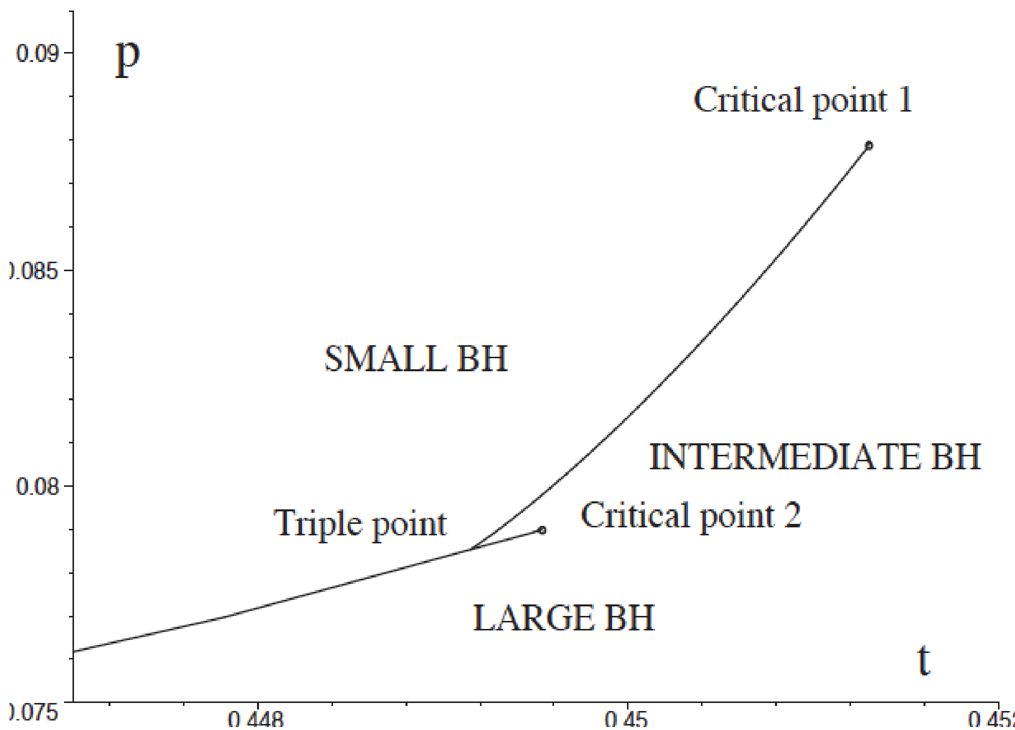
$$(d-3)M = (d-2)\frac{\partial M}{\partial A}A - 2\frac{\partial M}{\partial \Lambda}\Lambda$$

2nd-order Lovelock

Gauss – Bonnet

The existence of a triple point in d = 6 (charged BHs) is an exceptional case that has no counterpart in higher dimensions.

$\kappa = 1$



Two critical points and a triple point

The first order phase transition in the left low corner terminates at [0; 0].

Dynamical Cosmological constant: some implications

'Arxiv:
1510.02472
(A. Karch)
1404.5982
(C V. Johnson)

- Effective theory of non constant Lambda (inflation, quantum fluctuations)
- Isoperimetric inequalities (Conjecture for AdS Black Holes): "For a "black hole of given thermodynamic volume V, the entropy is maximised for Schwarzschild-AdS" [Ref. ArXiv:1012.2888]
- **Consistency** between First law and Smarr formula
- Thermodynamic machinery for study black holes phase transitions.
- Holographic Black Hole Chemistry

3^{rd} Order Lovelock Isolated critical point

A special case occurs when the parameter takes the value

$$\alpha = \sqrt{3}$$

- The system can be solved analytically
- A special isolated critical point characterized by **non-standard critical exponents** in the phase diagram of hyperbolic vacuum black holes.

$$\tilde{\alpha} = 0\,, \quad \tilde{\beta} = 1\,, \quad \tilde{\gamma} = 2\,, \quad \tilde{\delta} = 3$$

- In the Gibbs free energy: two swallowtails emerge, giving rise to two first-order phase transitions between small and large black holes.

Conclusions

The thermodynamic behaviour of the BB in Lovelock theory is *universal*:

- it does not depend on the higher order curvature terms
- only on the Einstein-Hilbert term, Λ and the matter fields content (in our case the EM field).

Consistently with the geometrical and thermodynamic picture,

- universality of η/s is lost in the UV but is restored in the IR
- possible existence of bounds lower than the KSS remains still open.

D=6 GB is the only dimension that admits triple points for charged BH

- For special tuned Lovelock couplings in the hyperbolic case a new type of isolated critical point, characterized by new critical exponents emerge.

Cosmological constant & its conjugate variable

Kastor, Ray, and Traschen, *Enthalpy and the Mechanics of AdSBlack Holes*, Class. Quant. Grav. 26 (2009) 195011, [arXiv:0904.2765].

- Identify: the cosmological constant with a thermodynamic pressure

$$P = -\frac{\Lambda}{8\pi G_N} = \frac{\hat{\alpha}_0}{16\pi G_N}$$

- Calculate its conjugate quantity, the "**thermodynamic volume**" of the black hole using the extended first law (Smarr formula):

$$(d-3)M = (d-2)TS - 2PV$$

Schwarzschild:

$$V = \frac{4}{3}\pi r_+^3$$

$$V = -\hat{\Psi}^{(0)} = \frac{\Sigma_{d-2}^{(\kappa)} r_+^{d-1}}{d-1}$$

$$\hat{\Psi}^{(k)} = -16\pi G_N \prod_{n=3}^{2k} (d-n)\Psi^{(k)}, \quad k \geq 2$$