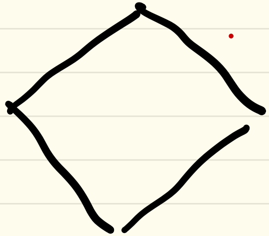
The saddle point description of Black Holes

Information Paradox

Minkowski

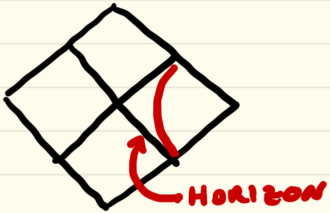
t, r



$$\langle \Omega | T\phi(x,t)\,\phi(x',t')|\Omega\rangle_{m=0} \sim \frac{1}{|x-x'|^2 - |t-t'|^2}$$

(Massless scalar field)

Rindler



τ, ρ

By a mere change of coords

$$\langle \Omega | T \phi(\tau, \rho) \phi(\tau' \rho') | \Omega \rangle \sim \frac{1}{1 - \cosh \tau \omega}$$

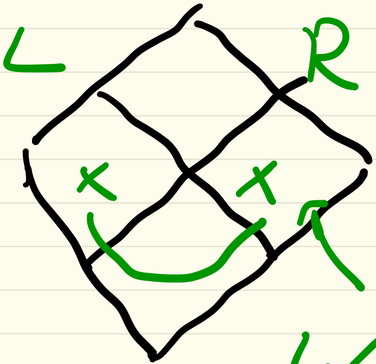
acc.

$$\Downarrow$$

$$\langle \phi \phi \rangle \sim e^{-\tau \omega} \quad !$$

Where is the information?

Rindler observer is simply blind to the "other side"



he/she misses L/R correlators!

However these correlators are accessible by others

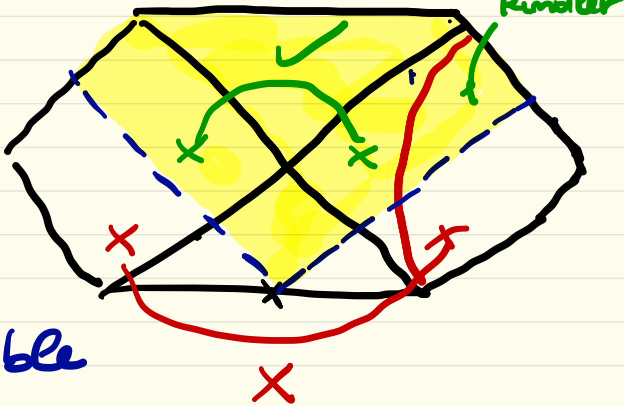
$\Downarrow$

total information = "Σ correlators"

$\Downarrow$

No info loss in Minkowski

BH



Accessible

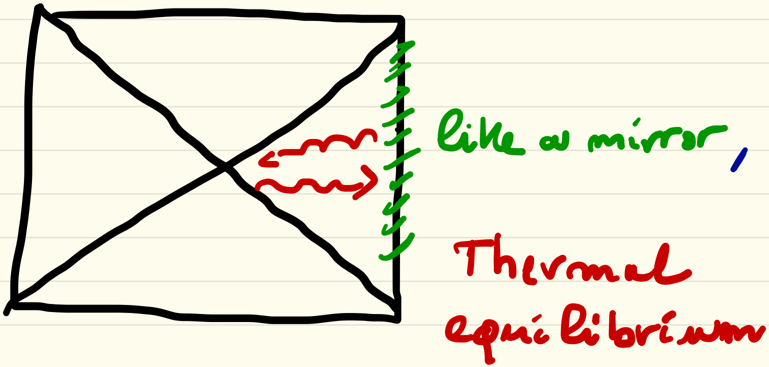
Information is lost for an "outsider"

Is information stored inside?

With evaporation is hard
(too much S)

Sharper problem

BH in AdS



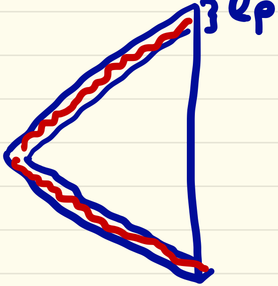
still $\langle \phi \phi \rangle \sim e^{-T/a}$

but CFT is unitary !

$\Downarrow$

Info must be accessible ?

But our calculation on BH background was certainly correct up to $\Delta x \sim \ell_p$



info $\propto S$
HUGE!!

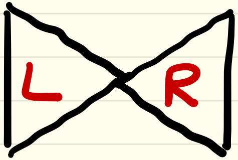
then all info is squeezed here

$\Downarrow$

Firewall!

Let us make it quantitative

Asymptotic AdS Rindler



At ε distance to the Horizon

$$\langle T^{e/L}_{tt} \rangle \sim -\frac{1}{\varepsilon^2} \Rightarrow$$ Firewall close to the Horizon

However modes correlating L/R also exist with a state

$$|\psi\rangle = \int d\omega \, f(\omega) \, b^\dagger_\omega \, |0\rangle_L \, |0\rangle_R$$

$$\Downarrow$$

$$\langle\psi| T_{tt} |\psi\rangle \sim \boxed{+} \frac{1}{\varepsilon^2}$$

this cancels the divergence!!

The Firewall in Rindler

$$\int Dg\, e^{iS} \sim \int Dg_{L,R}\, e^{iS}$$

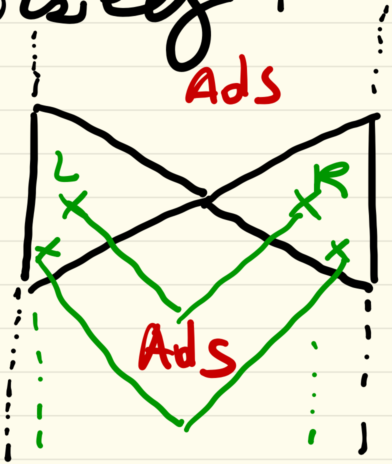
Resolution

$$\int Dg\, e^{iS} \sim \int Dg_{L,R} + \int Dg_{\text{entangled}}$$

$$\sim \int Dg_{\text{RINDLER}} + \int Dg_{\text{REST OF MINK.}}$$

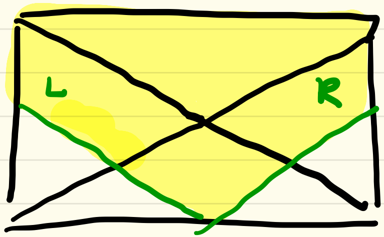
$$\sim \int Dg_{\text{MINK}} \quad !\ !$$

Rindler observer has discovered diffeomorphisms quantum mechanically!



All correlations are possible!

BH ?



Once again not all correlations can be taken

$\Downarrow$

Firewall

(Just as in Rindler)

$$\Downarrow$$

$$\langle OOS \rangle = \int Dg\, D\Phi\; O\, O\, e^{iS}$$

$$\neq \int Dg_{\substack{\text{CLASSICAL}\\ \text{SADDLE}}}\; D\Phi\; O\, O\, e^{iS}$$

We need something connecting left and right

The classical saddle point approx fails 2 times:

1) generates a firewall

2) is incompatible with the CFT

↳ CFT is suppose to know everything about the bulk.

But how can it get info from inside horizon? ($\sim$ info loss)

But we know saddle point approx is pretty good? ----

or not?

Wait may be there is a caveat ----

Saddle point approx

$$\int f(t)\, e^{-A(t)}\, dt \sim f(t_0)\, e^{-A(t_0)}$$

where $\left. A'(t) \right|_{t_0} = 0$

assuming $f(t_0) \neq 0$!

if not ----

$$\int dt\, g(t)\, e^{-A(t)} \qquad (g(t_0)=0)$$

$$\sim e^{A(t_0)} \left(-\frac{g''}{4A''}\Big|_{t_0} + \frac{g' A'''}{8A'^2}\Big|_{t_0} + \cdots \right)$$

(Note in addition that A'' should not vanish)

In gravity

$$\langle \phi\phi \rangle \sim \int Dg\, D\phi\, \phi\phi\, e^{iS_g} e^{iS_\phi}$$

$$\sim \int D\phi\, \underbrace{\phi\phi\, e^{iS_\phi(g_0)}}_{\rightsquigarrow f(t_0)}$$

where $\left.\frac{\delta S_g}{\delta g}\right|_{g_0} = 0$

assuming $\int D\phi\, \phi\phi\, e^{iS_\phi(g_0)} \neq 0\,!$

but in fact for a BH we know that asimptotically

$\langle \phi \phi \rangle \to 0$! $(t \to \infty)$

$\Downarrow$

Saddle point breaks down!

2 options

1) complex saddle points contribute largely

2) next to leading orders in saddle point dominate

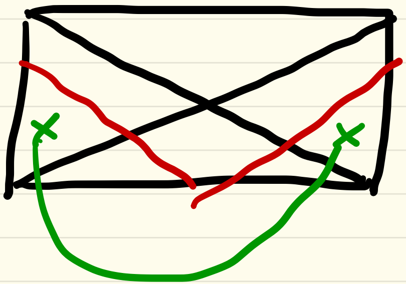
Effectively: remember we need to be able to look inside the horizon

a) New saddle points do not have horizon (Hawking 2000)

b) Next to leading saddle point

$\Rightarrow$ non-locality ....

Horizons only causally disconnect regions for local operators



BH

we can correlate if the scalar is non-local.

Is there a third option? Namely $O(e^{-S})$ correction to the Hawking spectrum?

(CG, SARKAR, 15) NO!

Although a far observers sees a unitary spectrum we showed explicitly a Firewall!

Briefly:

The correct unitarization of the Hawking spectrum goes

$$\mathcal{O}\left(g^{tt} e^{-S}\right) \xrightarrow[\text{at horizon}]{} \infty \; !$$

thus expansion breaks down

Case a: New saddle points

CG '13

2D BH does not evaporate

$\Downarrow$

In principle we do not need extra saddle points

However even here for a BH they are necessary ---

2D gravity

$$A = \int d^2x \sqrt{-g}\,(R - 2\Lambda)$$

$$\frac{\delta A}{\delta g^{\mu\nu}} = 0 = \Lambda g_{\mu\nu} \qquad \Lambda \to 0$$

However suppose we have a CFT for example a massless scalar

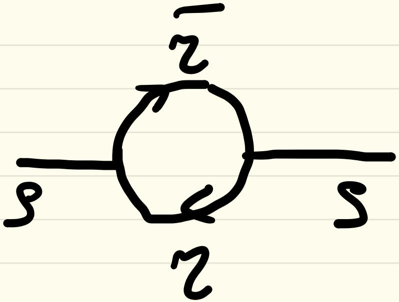
$$A_\phi = \int d^2x \sqrt{-g}\, \phi \Box \phi$$

$$ds^2 = -e^{2\rho} dx^+ dx^-$$

$$Z = \int D\phi \; e^{\int \phi \Box \phi} \rightarrow \det \sqrt{\Box}$$

$$\sim \int D\eta \, D\bar{\eta} \; e^{\int e^{-2\rho} \bar{\eta} \partial^2 \eta}$$

$$e^{2\rho} = 1 + 2\rho + \dots$$



$$\sim \rho \, \partial^2 \rho$$

So the effective action

$$\sim \int d^2x \; e^{2\rho} \, \partial^2 e^{2\rho} \sim \int d^2x \sqrt{-g} \, R \, \frac{1}{\square} \, R$$

$$\Downarrow$$

Liouville theory

then at one loop in matter

Gravity is no longer trivial

$$\Lambda \sim \hbar R$$

in particular solving

$$\Lambda g_{\alpha\beta} \sim \langle T^{\phi}_{\alpha\beta} \rangle$$

$$e^{2S} \propto \begin{cases} a) & \sinh^{-2}\left[\frac{1}{2}(x^+ - x^-)\sqrt{q}\right] \\ b) & -\cosh^{-2}\left[\frac{1}{2}(x^+ - x^-)\sqrt{q}\right] \end{cases}$$

q defines vacuum ....

a) $\Rightarrow$ $x^+ - x^-$ is spacelike

b) $\Rightarrow$ $x^+ - x^-$ timelike

we take (a)

BH

$$-e^{2S}\, dx^+ dx^- \to -f(x)\,dt^2 + \frac{dx^2}{f(x)}$$

$$f(x) = \frac{x^2}{\ell^2} \pm 2\mu x - 1$$

of course $f(x) > 0$

we analitically extend for $f(x) < 0$

$$f(x) = \frac{x^2}{\ell^2} - 2\mu|x| - 1$$

Now μ is a BH mass and inside the horizon $x < x_0$ $(f(x_0) = 0)$

we get the second (b) solution

$$e^{2\rho} < 0$$

BH

time-like
Liouville

space like
Liouville

the 2D BH glues 2 Liouville theories

Now we consider $\int Dg \sim \int D\varsigma$

Primary operators $V_\alpha = e^{2\alpha\varsigma}$

. Spacelike (outside horizon)

$$\langle V_\alpha V_\beta \rangle = \langle V_\alpha V_\beta \rangle_{\text{SADDLE}}$$

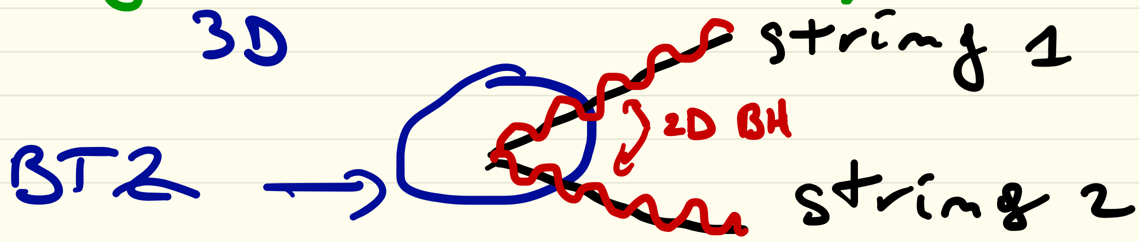
. timelike (inside) [Witten et al.]

$$\langle V_\alpha V_\beta \rangle = \sum_{\text{complex saddle}} \langle V_\alpha V_\beta \rangle$$

$\Rightarrow$ in 2D the BH is described by 2 saddle points!

Although is an IR object is very quantum.

Holography (Cr, Procopio, 06)



But! timelike Liouville is not unitary!

$\Downarrow$

Interior is a condensate of exterior dof

(Gia's point of view)

[Polkorniakov]

indeed the interior is mapped on 2 interacting Bose-Einstein condensates

The Saga of 2's

- 2 BEC, map of field theories
- 2 saddles, direct computation
- 2 strings, holography

Case b 3D BH, BTZ

(CCG, work in progress)

after long time

$\langle \partial \partial \rangle \to 0$

so we need to go next to leading saddle.

$$Z \sim \int D\phi\, Dg\; e^{\underbrace{-\frac{1}{\hbar}\int \mathcal{L}_G}_{G/\hbar}}\; e^{\underbrace{-\frac{1}{\hbar}\int \mathcal{L}_\phi}_{\Psi/\hbar}}$$

saddle $\quad \dfrac{\delta G+\Psi}{\delta g} = 0 \Rightarrow g_0$

$$g = g_0 + \delta g \sqrt{\hbar}$$

$$\frac{G+\Psi}{\hbar} \sim \frac{G_0+\Psi_0}{\hbar} + \delta g \left(\frac{\delta^2 G}{\delta g\, \delta g} + \frac{\delta^2 \Psi}{\delta g\, \delta g} \right) \delta g$$

We have a gravity Gaussian integral in δg

$$\Downarrow$$

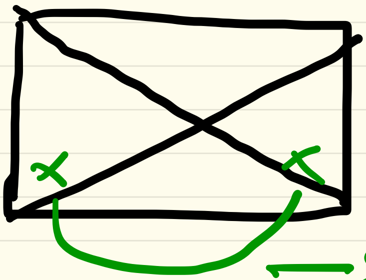
$$\delta g \underbrace{\left[M_p (\Box - 7\Lambda) - (\partial\phi)^2 \right]}_{\Delta} \delta g$$

so the effective action for ϕ (after long time) $\sim \det \sqrt{\Delta}$

$$\mathcal{L}_{eff} \sim \int d^3x_1 \ldots\ldots d^3x_n \; G(x_1 - x_2)(\partial\phi)^2_{x_2}$$

$$\ldots\ldots \underbrace{G(x_n - x_1)}_{\text{Green's in BTZ with imaginary mass!}}(\partial\phi)^2_{x_1}$$

Non-local!



$\longrightarrow$ correlations ok!

We know from AdS/CFT that BH is a CFT at temp. T

At initial time termal noise dominates, but after long time the system is periodic

Semi-classical BHs
missed the periodicity
(info loss)

But: in the next to leading
we have now a periodicity
in G related to the
imaginary mass ....
may be in the right track ...

Summary

- BH are IR but cannot be described by classical saddles
- Consider complex saddles and next to leading saddles
- Effective description may be seen as a condensate ...

Thanks