Emerging geometry of corpuscular black holes

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Outline

1. Motivation: gravitational radius and mass threshold
2. Horizon wave-function (HWF): definition and scope
3. Probability of BH formation: a simple example
4. Generalised Uncertainty Principle (GUP)
5. Corpuscular BHs
6. Conclusions

The clash of two lengths: GR vs QM

GR: Schwarzschild radius

- $g^{ij}\nabla_i r \nabla_j r = 0 \to$ trapped surface
- Very hard to define energy $\subset$ generic closed surface
- Spherical symmetry $\to$ MS mass

$$m(t,r) = 4\pi \int_0^r \rho(t,\bar{r})\bar{r}^2 \mathrm{d}\bar{r}$$

- New def. of trapped surface $r = R_{\mathrm{H}}(t,r) = 2\frac{\ell_{\mathrm{P}}}{m_{\mathrm{P}}}\, m(t,r)$
- If the system is static $\to$ horizon
- Problem! No mass threshold

QM: Compton Wave-length

- Particle's position and momentum are uncertain
- Cut-off in spatial localisation $\lambda_m = \frac{\ell_{\mathrm{P}} m_{\mathrm{P}}}{m}$
- Nature is Quantum!
- $R_{\mathrm{H}} \geq \lambda_m$, otherwise "screened" (*i.e.* negligible)
- BH mass must satisfy $m \geq m_{\mathrm{P}}$

A consistent quantum theory of gravity should view R_{H} and λ_m on equal grounds

Horizon Wave Function

see R.Casadio, A.G. and O.Micu, Int. J. Mod. Phys. D **25** (2016) no.02, 1630006

The idea (Casadio, 2013):

- Source decomposed as $|\,\psi_{\rm S}\,\rangle = \sum_E C(E)|\,\psi_E\,\rangle$, at rest in the given ref. frame
- $\forall$ energy eigenstate...

$$\hat{H}|\,\psi_E\,\rangle = E|\,\psi_E\,\rangle$$

...$\exists$ a Schwarzschild radius

$$\left(\hat{H} - \frac{m_{\rm P}}{2\ell_{\rm P}}\hat{R}_{\rm H}\right)|\,\psi_E\,\rangle = 0 \quad \Longrightarrow \quad r_{\rm H} = 2\frac{\ell_{\rm P}}{m_{\rm P}}E$$

- Horizon not exactly localised (fuzzy) & state described by

$$\psi_{\rm H}(r_{\rm H}) \propto C\left(\frac{m_{\rm P}\, r_{\rm H}}{2\ell_{\rm P}}\right)$$

- Spherical symmetric expectation values

$$\langle\,\hat{O}\,\rangle = 4\pi\int_0^\infty \psi_{\rm H}^*\,\hat{O}\,\psi_{\rm H}\,r_{\rm H}^2\,{\rm d}r_{\rm H}$$

Horizon Wave Function

Massive spin-less neutral particle:

- A Gaussian wave-packet, $\Delta = m_{\mathrm{P}}\ell_{\mathrm{P}}/\ell$, ...

$$\psi_{\mathrm{S}}(r) = \frac{e^{-\frac{r^2}{2\ell^2}}}{(\ell\sqrt{\pi})^{3/2}} \quad \underset{\mathcal{F}}{\Longrightarrow} \quad \widetilde{\psi}_{\mathrm{S}}(p) = \frac{e^{-\frac{p^2}{2\Delta^2}}}{(\Delta\sqrt{\pi})^{3/2}}$$

- ... through the flat mass shell $p^2 = E^2 - m^2$...

... gives the HWF

$$\psi_{\mathrm{H}}(r_{\mathrm{H}}) \;=\; \frac{1}{4\ell_{\mathrm{P}}^3}\sqrt{\frac{\ell^3}{\pi\,\Gamma\left(\frac{3}{2},\frac{m^2}{\Delta^2}\right)}}\,\Theta(r_{\mathrm{H}} - R_{\mathrm{H}})\,\exp\left\{-\frac{\ell^2\, r_{\mathrm{H}}^2}{8\ell_{\mathrm{P}}^4}\right\}$$

P_{BH}

What is the probability of finding the particle inside r_H?

$$P_S(r < r_{\mathrm{H}}) = 4\pi \int_0^{r_{\mathrm{H}}} \mathrm{d}r \; r^2 \; |\psi_S(r)|^2$$

What is the probability density that the sphere $r = r_H$ is a horizon?

$$\mathcal{P}_{\mathrm{H}}(r_{\mathrm{H}})\mathrm{d}r_{\mathrm{H}} = 4\pi \; r_{\mathrm{H}}^2 \; |\psi_{\mathrm{H}}(r_{\mathrm{H}})|^2 \mathrm{d}r_{\mathrm{H}}$$

Probability that particle is a black hole $=$
probability that particle is inside r_{H} **AND** that r_{H} is the horizon **FOR ALL** possible values of r_H

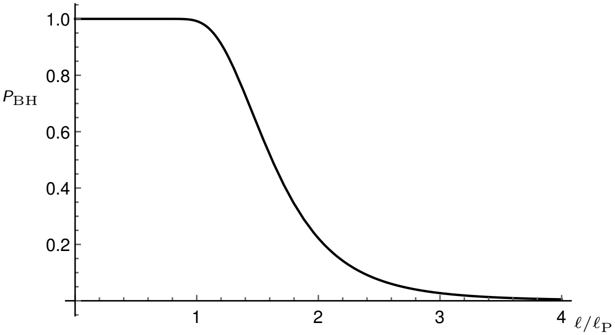
$$P_{\mathrm{BH}} = \int_0^{\infty} \mathcal{P}_{\mathrm{H}}(r_{\mathrm{H}}) P_S(r < r_{\mathrm{H}}) \mathrm{d}r_{\mathrm{H}}$$

$P_{\rm BH}$

- In general, we have to compute $P_{\rm BH}$ numerically

Important remarks:

- Maximum $P_{\rm BH} \to \ell \lesssim \ell_{\rm P}$
- Large fall-off when $\ell \gg \ell_{\rm P}$



- **Large chance of BHs** $\Rightarrow$ $\ell \sim \ell_{\rm P}$ & $m \sim m_{\rm P}$

GUP and minimum length

- Max. localisation: $\ell = \lambda_m$
- As usual $\langle \hat{r}_{\rm H} \rangle \propto R_{\rm H}$ and $\langle \hat{r}_{\rm H}^2 \rangle \propto R_{\rm H}^2$
- The uncertainty $\Delta r_{\rm H} \propto R_{\rm H} \sim \Delta p$

The total uncertainty is a linear combination

$$\frac{\Delta r}{\ell_{\rm P}} = \frac{C_{\rm QM}}{\Delta p} + \xi C_{\rm H} \Delta p$$

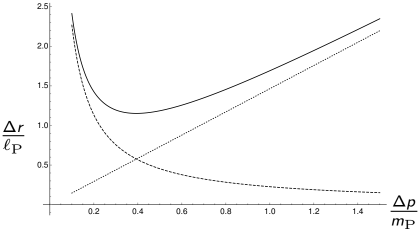
Problem...

fluctuations are too large:

$\Delta r_{\rm H} \simeq \langle \hat{r}_{\rm H} \rangle$

R.Casadio, A.G., O.Micu and A.Orlandi,

PRD 90 (2014) 084040



- **A single particle is not a good BH candidate**

Corpuscolar Model of BH

The idea (Dvali & Gomez, 2012):

- Confinement $\Longrightarrow$ gravitational interaction $V_{\rm N} \simeq -\frac{\ell_{\rm P}}{m_{\rm P}}\frac{M}{r}$ (Newton) with total energy $M = Nm$
- λ_m as characteristic lengthscale $\Longrightarrow$ effective mass $m = m_{\rm P}\ell_{\rm P}/\lambda_m$
- Coupling constant & average potential energy per constituent

$$\alpha = -\frac{V_{\rm N}(\lambda_m)}{N} \simeq \frac{m^2}{m_{\rm P}^2}\ , \qquad U \simeq m V_{\rm N}(\lambda_m) \simeq -N\,\alpha\,m$$

- BH made up by a large number of constituents all in the same quantum state $\Longrightarrow$ **Bose-Einstein Condensate** (BEC)

R.Casadio, A.G., O.Micu and A.Orlandi, Entropy 17 (2015) 6893-6924

Key features:

- Constituents are "marginally bound" $\to \alpha \simeq 1/N$
- Masses and horizon size are quantised

$$m \simeq \frac{m_{\mathrm{P}}}{\sqrt{N}} \ , \quad M = Nm \simeq \sqrt{N} m_{\mathrm{P}} \ ,$$

$$R_{\mathrm{H}} = 2\ell_{\mathrm{P}} \frac{M}{m_{\mathrm{P}}} \simeq 2\sqrt{N}\ell_{\mathrm{P}}$$

- Hawking radiation $\simeq$ BEC **quantum depletion** through scattering
- Emission rate at first order ($2 \to 2$ scattering)

$$\Gamma \sim \frac{1}{N^2} N^2 \frac{1}{\sqrt{N}\ell_{\mathrm{P}}} \implies \dot{N} = -\Gamma = -\frac{1}{\sqrt{N}\ell_{\mathrm{P}}} + \mathcal{O}\left(\frac{1}{N}\right)$$

- Hawking flux:

$$\dot{M} = m_{\mathrm{P}} \frac{\dot{N}}{\sqrt{N}} = -\frac{m_{\mathrm{P}}^3}{\ell_{\mathrm{P}}} \frac{1}{M^2} \implies T_{\mathrm{H}} \simeq \frac{m_{\mathrm{P}}^2}{8\pi M} \sim m$$

A simple example

R.Casadio, A.G. and A.Orlandi, PRD 91 (2015) 124069

Single-particle wave-function = superposition of energy eigenstates

- discrete ground state $|\, m\,\rangle$: $\hat{H}|\, m\,\rangle = m|\, m\,\rangle$
- gapless continuous spectrum $\hat{H}|\, \omega_i\,\rangle = \omega_i|\, \omega_i\,\rangle$ arranged in a Planckian distribution

$$|\, \psi^{(i)}\,\rangle = \frac{\mathcal{N}_{\rm H}}{m^{3/2}} \int_m^\infty {\rm d}\omega_i \, \frac{\omega_i - m}{\exp\{(\omega_i - m)/m\} - 1} \, |\, \omega_i\,\rangle$$

Single particle wavefunction

$$|\, \Psi_{\rm S}^{(i)}\,\rangle = \frac{|\, m\,\rangle + \gamma_1 |\, \psi^{(i)}\,\rangle}{\sqrt{1+\gamma_1^2}}$$

- Many-body description $\rightarrow$ symm. product of N single-particle WFs
- Approximation: $C(E > M) \simeq \gamma \frac{\mathcal{N}_{\mathrm{H}}}{\sqrt{m}} \frac{(E-M)/m}{\exp\{(E-M)/m\}-1}$

Result:

BEC is effectively the single-particle state

$$|\,\Psi_S\,\rangle \simeq \frac{|\,M\,\rangle + \gamma|\,\psi\,\rangle}{\sqrt{1+\gamma^2}}$$

$$|\,\psi\,\rangle = \frac{\mathcal{N}_{\mathrm{H}}}{\sqrt{m}} \frac{(E-M)/m}{\exp\{(E-M)/m\}-1}\,|\,E\,\rangle$$

$$\hat{H}|\,M\,\rangle = M|\,M\,\rangle\ , \qquad \hat{H}|\,E\,\rangle = E|\,E\,\rangle\ .$$

- BEC BH effectively looks like one particle of very large mass M, in a superposition of "Planckian hair" states
- Also works outside the perturbative regime *i.e.*, when $\gamma \simeq 1$

Thermodynamics

- Expectation values and thermodynamical functions are easy to obtain, $\beta = T_{\mathrm{H}}^{-1}$

Statistical canonical entropy

$$S(\beta) = \beta^2 \frac{\partial F(\beta)}{\partial \beta} = \frac{A_{\mathrm{H}}}{4\ell_{\mathrm{P}}^2} - \frac{\mathcal{K}}{2}\gamma^2 \log\left(\frac{A_{\mathrm{H}}}{16\pi\ell_{\mathrm{P}}^2}\right) .$$

Bekenstein-Hawking + log corrections!

Specific heat

$$C_V(\beta) = -\beta^2 \frac{\partial\langle \hat{H} \rangle}{\partial \beta} = -m_{\mathrm{P}}\beta^2 + \frac{\mathcal{K}\gamma^2}{m_{\mathrm{P}}}$$

- Vanishes for $\beta \simeq \gamma/m_{\mathrm{P}} \Rightarrow N_c \sim \gamma^2$
- $\gamma \sim N \Longrightarrow N_c \sim 1$, no more quanta to emit

...And what has HWF to say?

Expectation value and uncertainty of $\hat{r}_H$

$$\langle \hat{r}_H \rangle = R_\mathrm{H} \left[1 + \frac{3\gamma^2}{N} \mathcal{N}_\mathrm{H}^2 \left(6\zeta(3) - \frac{\pi^4}{15} \right) \right] + O(\gamma^4) \ ,$$

$$\Delta r_\mathrm{H} = \sqrt{\left| \langle \hat{r}_H^2 \rangle - \langle \hat{r}_H \rangle^2 \right|} \simeq 1.27\, \gamma \, \frac{R_\mathrm{H}}{\sqrt{N}}$$

- Well-defined GR limit: $N \gg 1$

$$\langle \hat{r}_H \rangle \xrightarrow[N \gg 1]{} R_\mathrm{H} \ , \qquad \Delta r_\mathrm{H} \xrightarrow[N \gg 1]{} 0$$

- $\langle \hat{r}_H \rangle \gtrsim R_\mathrm{H} \rightarrow$ QM corrections mimic **backscattering**
- Smaller depletion rate

$$\Gamma \sim \frac{1}{\langle \hat{r}_H \rangle} \simeq \frac{1}{\sqrt{N} \ell_\mathrm{P}} \left[1 - \frac{3\gamma^2}{N} \mathcal{N}_\mathrm{H}^2 \left(6\zeta(3) - \frac{\pi^4}{15} \right) \right]$$

- Again, **flux will stop** for $N_c \sim \gamma^2$

Conclusions

- A semiclassical approach is appropriate and required to study quantum aspects of gravity (especially outside a mere conceptual arena)
- HWF allows to compute the probability that an extended massive object may lie inside its own event horizon
- It reproduces the standard GUP, but 1 "Planckian-size" particle alone is not a good candidate
- A realistic model is described by many gravitons in a BEC, and the geometrical properties of gravity emerge in an effective way
- The standard thermodynamical properties of quantum BHs are implied by both corpuscular model & HWF

Thank you for your attention!