Manifestly diffeomorphism invariant exact RG

II Flag Workshop "The Quantum & Gravity"

06/06/16

Tim Morris,

Physics & Astronomy,

University of Southampton, UK.

TRM & Anthony W.H. Preston, arXiv:1602.08993, JHEP ? (2016) ?

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GR:

- $g_{\mu\nu}$ is a field

QM:

- which must be consistent with QM

↓

- Effective (quantum) field theory
- ...from 29 Gpc to 10^{19} GeV ($\times 10^{60}$)

Even classical GR

Even classical GR: back-reaction

Universe is **not** homogeneous!

$$\frac{\delta\rho_{\rm Earth}}{\langle\rho\rangle} \sim 10^{31}$$

$$\langle T_{\mu\nu}\rangle = T^{(0)}_{\mu\nu} \qquad\qquad \langle g_{\mu\nu}\rangle = g^{(0)}_{\mu\nu}$$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad \langle h_{\mu\nu}\rangle = 0$$

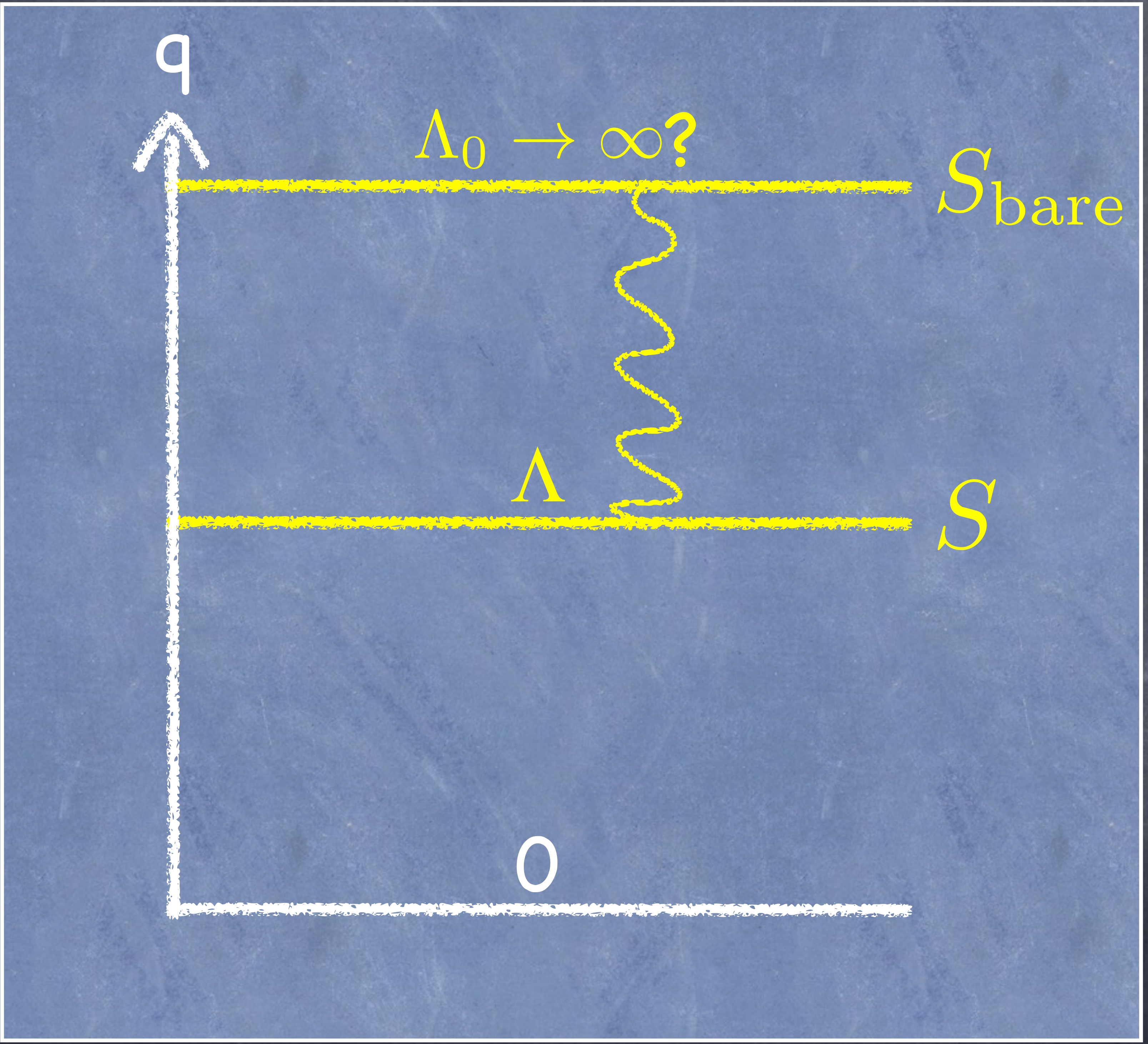
$$\kappa := 8\pi G$$

$$R^{(0)}_{\mu\nu} - \frac{1}{2} g^{(0)}_{\mu\nu} R^{(0)} = \kappa T^{(0)}_{\mu\nu} + \kappa t^{(0)}_{\mu\nu}$$

Could this give an effective cosmological const?
& answer "why now?"

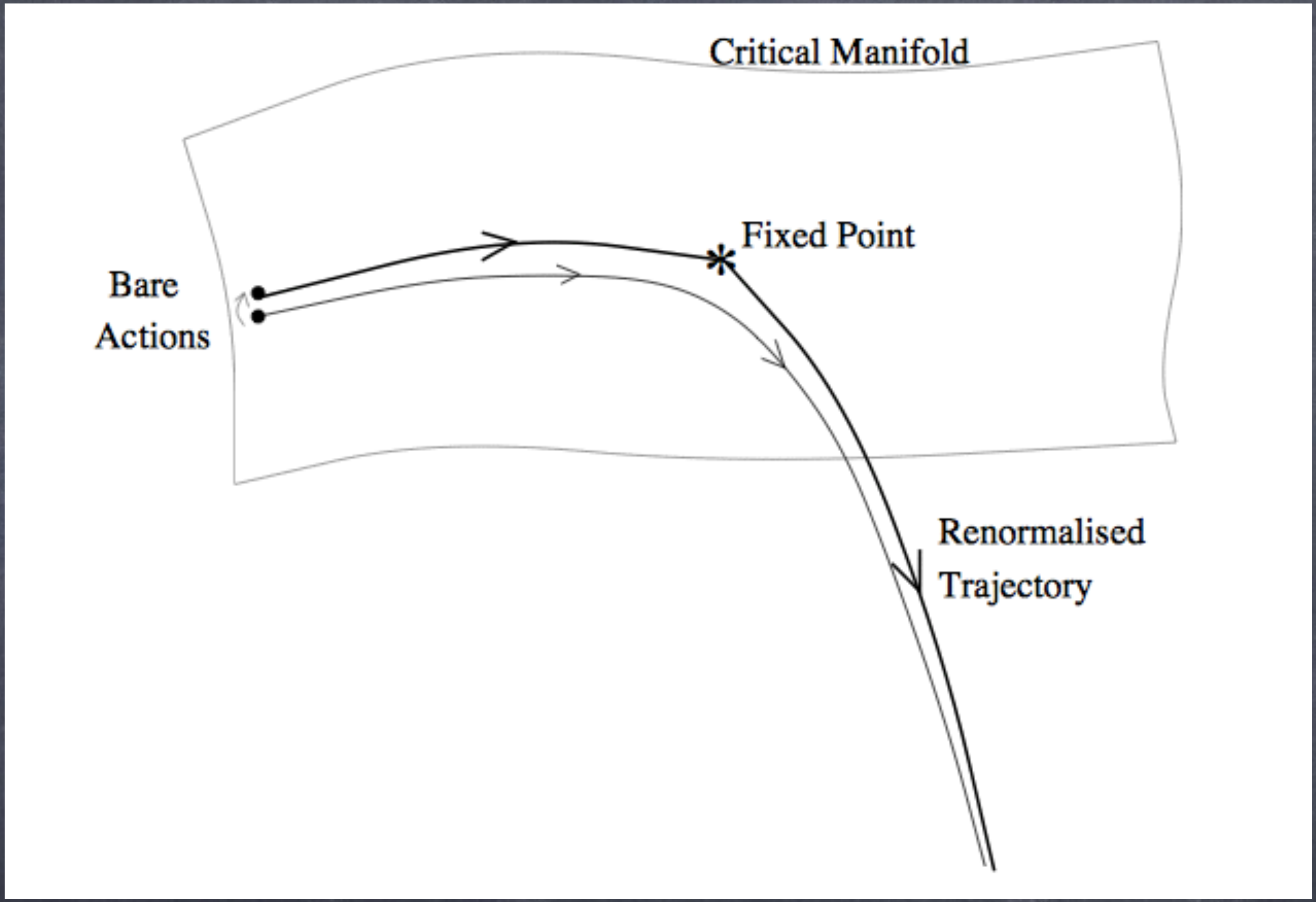
S. Räsänen, D.J. Schwarz, C. Clarkson, G. Ellis, T. Clifton, T. Buchert, S.R. Green & R.M. Wald; TRM & A.W.H. Preston; N.J. Evans, TRM & M Scott; A.W. Preston; ...

Wilsonian RG



$$\mathcal{Z} = \int \mathcal{D}\varphi \; \mathrm{e}^{-S[\varphi]} = \int \mathcal{D}\varphi_0 \; \mathrm{e}^{-S_{\text{bare}}[\varphi_0]}$$

Wilsonian RG

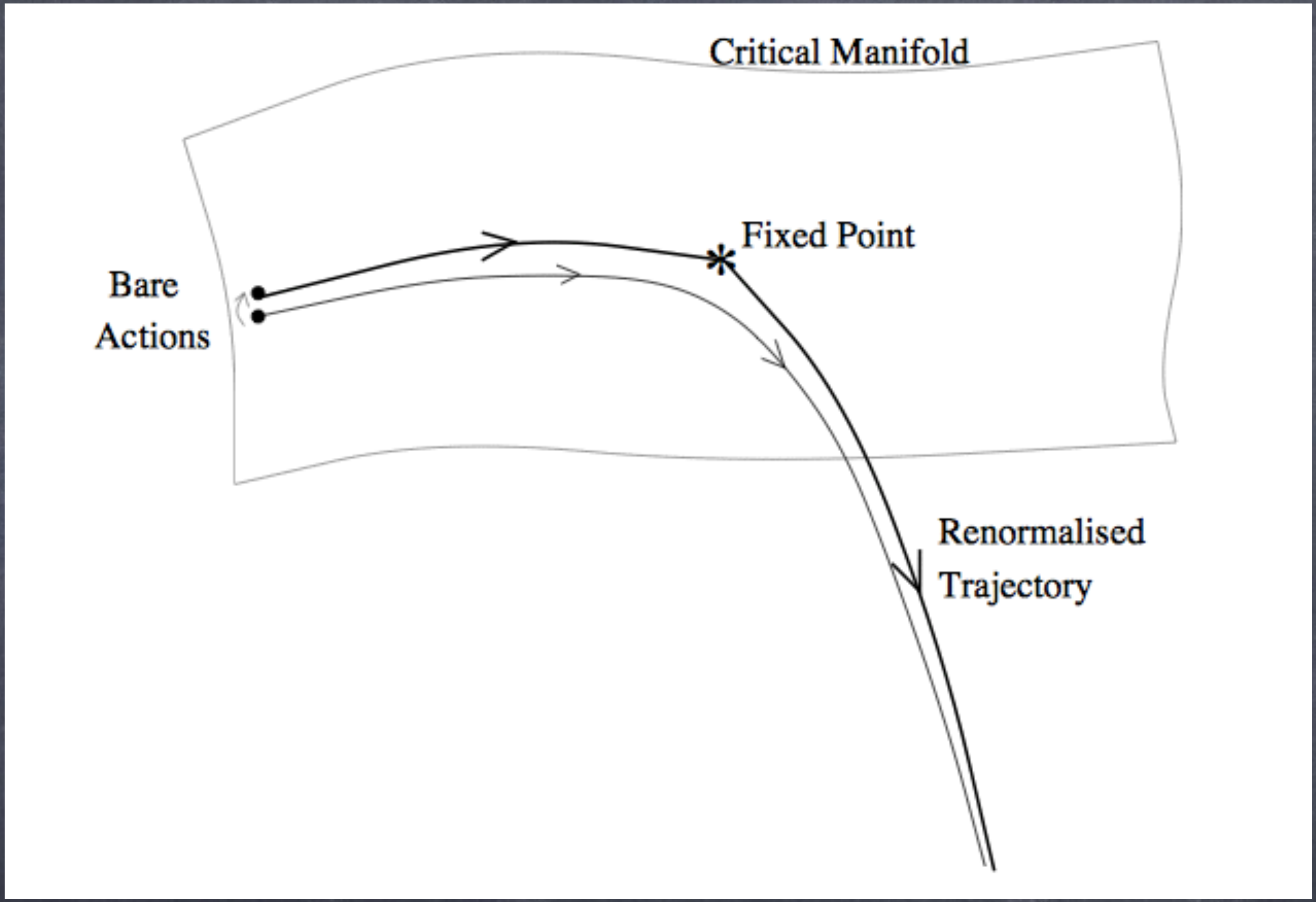


Theory space...

$$S = \int d^4x\,\sqrt{g}\,\{g_0 + g_2(-2R) + g_{4,1}R^2 + g_{4,2}R^{\mu\nu}R_{\mu\nu} + g_{6,1}R^3 + g_{6,2}RR^{\mu\nu}R_{\mu\nu} + \cdots$$

$$\frac{\lambda}{8\pi G} \qquad \frac{1}{\tilde{\kappa}} = \frac{1}{32\pi G}$$

Wilsonian RG



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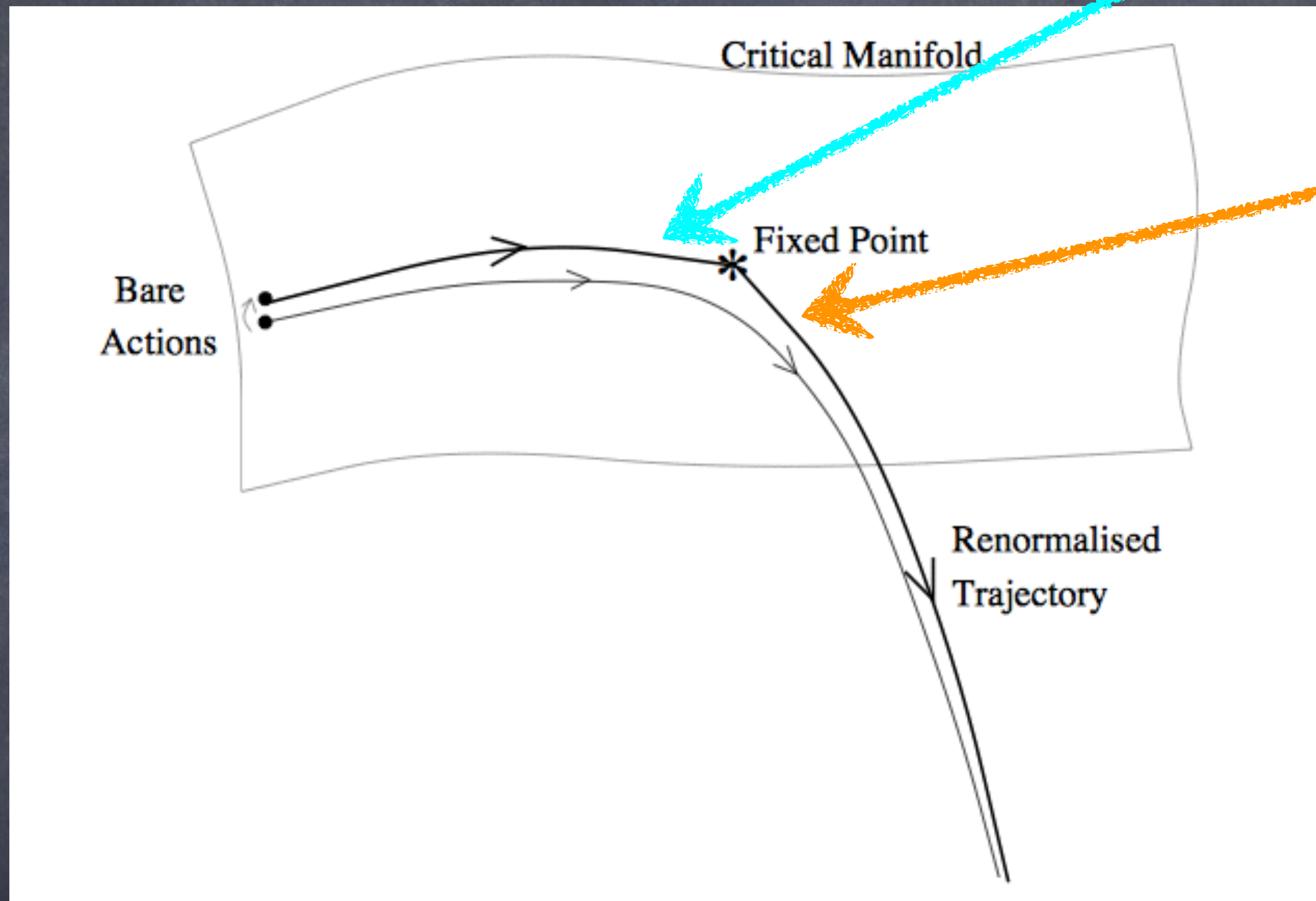
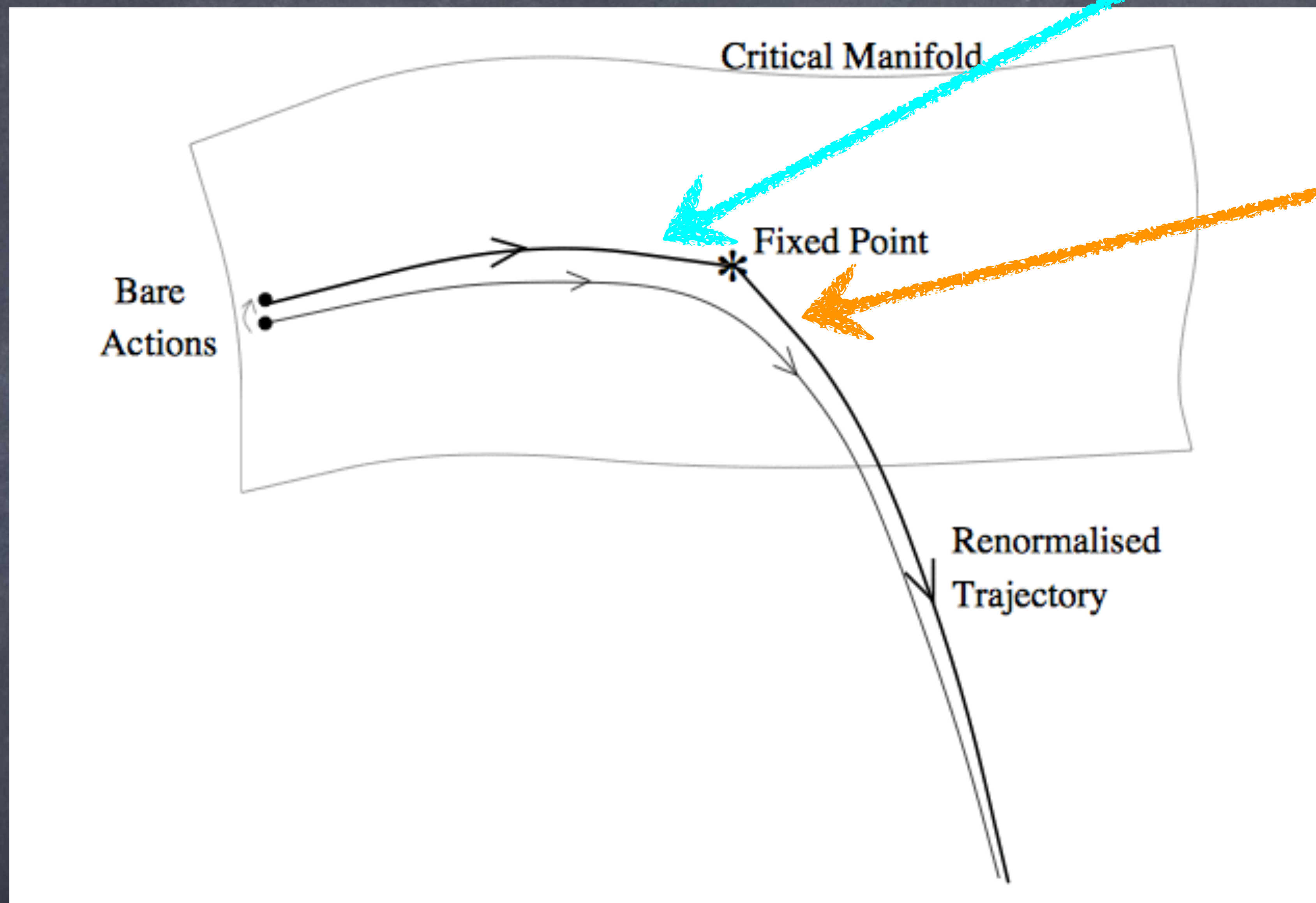
$$\frac{\lambda}{8\pi G} \qquad \frac{1}{\tilde{\kappa}} = \frac{1}{32\pi G} \qquad g_{2n,\alpha_n} \sim \Lambda^{4-2n}$$

Wilsonian RG



G is irrelevant about Gaussian FP, but
effective field theory over 60 orders of magnitude

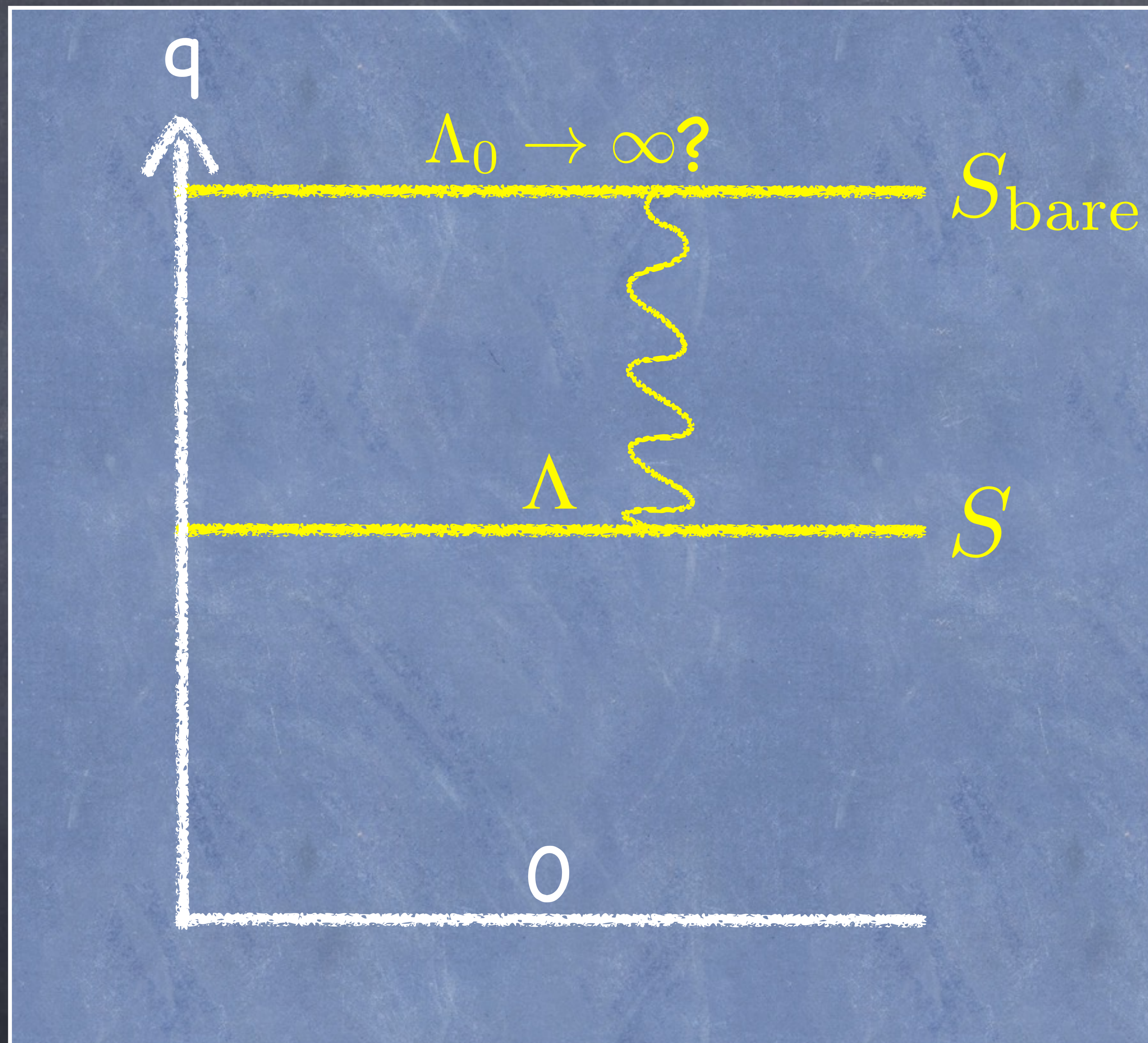
Wilsonian RG



G is irrelevant about Gaussian FP, but
effective field theory over 60 orders of magnitude

Asymptotic safety: does a non-perturbative FP exist?

Wilsonian RG?



$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\ \mathrm{e}^{-S[g]} = \int \mathcal{D}g_{0\mu\nu}\ \mathrm{e}^{-S_{\rm bare}[g_0]}$$

- What does q mean? Modes for what metric?
- Gauge fixing requires background $\bar{g}_{\mu\nu}$
- Two metric theory except on shell?
- Diffeomorphism invariance only on shell?
- What does on shell mean for Wilsonian RG?

Manifestly diffeomorphism invariant classical exact RG

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\ \mathrm{e}^{-S[g]} = \int \mathcal{D}g_{0\mu\nu}\ \mathrm{e}^{-S_{\mathrm{bare}}[g_0]}$$

TRM & Anthony W.H. Preston, arXiv:1602.08993, JHEP … (2016) …

- Scalar field theory
- (non-Abelian) gauge theory
- Gravity

Kadanoff blocking

J.I. Latorre & TRM, Exact scheme independence, JHEP 11 (2000) 004;
S. Arnone, TRM & O.J. Rosten, Fields Inst. Commun. 50 (2007) 1

$$e^{-S[\varphi]} = \int \mathcal{D}\varphi_0 \, \delta\left[\varphi - b\left[\varphi_0\right]\right] e^{-S_{\mathrm{bare}}[\varphi_0]} .$$

The blocking functional is, in turn, a scalar field with a position argument. A simple linear example of a blocking functional in a D-dimensional field theory is

$$b[\varphi_0](x) = \int_y B(x-y)\varphi_0(y), \tag{2.2}$$

where $B(z)$ is a kernel that provides a smooth infrared cutoff such that $B(z)$ decays rapidly towards zero once $|z|\Lambda > 1$. This allows us to integrate out the higher momentum modes while keeping our effective action as an expansion in local operators.

$$\mathcal{Z} = \int \mathcal{D}\varphi \; \mathrm{e}^{-S[\varphi]} = \int \mathcal{D}\varphi_0 \; \mathrm{e}^{-S_{\mathrm{bare}}[\varphi_0]}$$

Kadanoff blocking

$$e^{-S[\varphi]} = \int \mathcal{D}\varphi_0 \; \delta\left[\varphi - b\left[\varphi_0\right]\right] e^{-S_{\text{bare}}[\varphi_0]}.$$

$$\Lambda \frac{\partial}{\partial \Lambda} e^{-S[\varphi]} = -\int_x \frac{\delta}{\delta\varphi(x)} \int \mathcal{D}\varphi_0 \; \delta\left[\varphi - b\left[\varphi_0\right]\right] \Lambda \frac{\partial b[\varphi_0](x)}{\partial \Lambda} e^{-S_{\text{bare}}[\varphi_0]}$$

$$= \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi(x) e^{-S[\varphi]}\right)$$

$$\mathcal{Z} = \int \mathcal{D}\varphi \; \mathrm{e}^{-S[\varphi]} = \int \mathcal{D}\varphi_0 \; \mathrm{e}^{-S_{\text{bare}}[\varphi_0]}$$

$$\dot{S} = \int_x \Psi(x) \frac{\delta S}{\delta \varphi(x)} - \int_x \frac{\delta \Psi(x)}{\delta \varphi(x)}$$

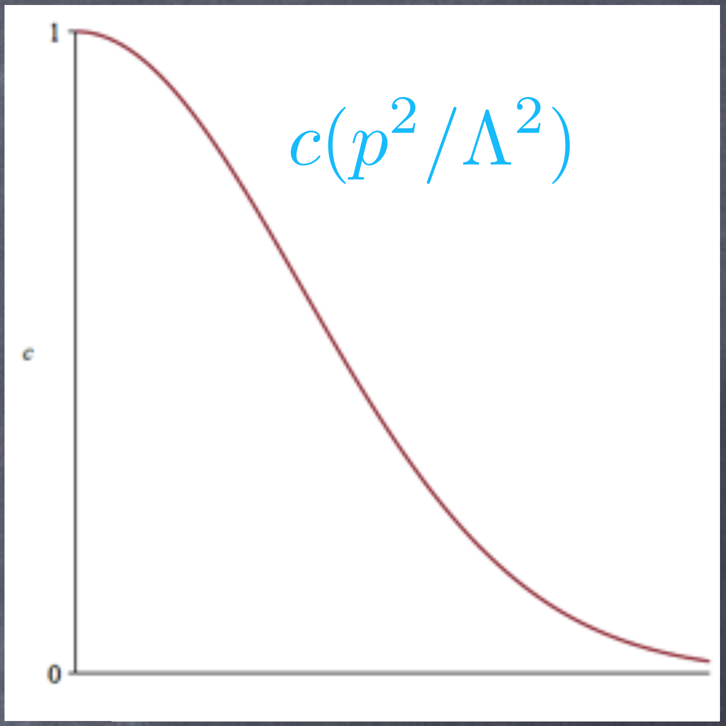
General exact RG flow eqn

$$\dot{S} = \int_x \Psi(x) \frac{\delta S}{\delta \varphi(x)} - \int_x \frac{\delta \Psi(x)}{\delta \varphi(x)}$$

Wilson/Polchinski flow eqn

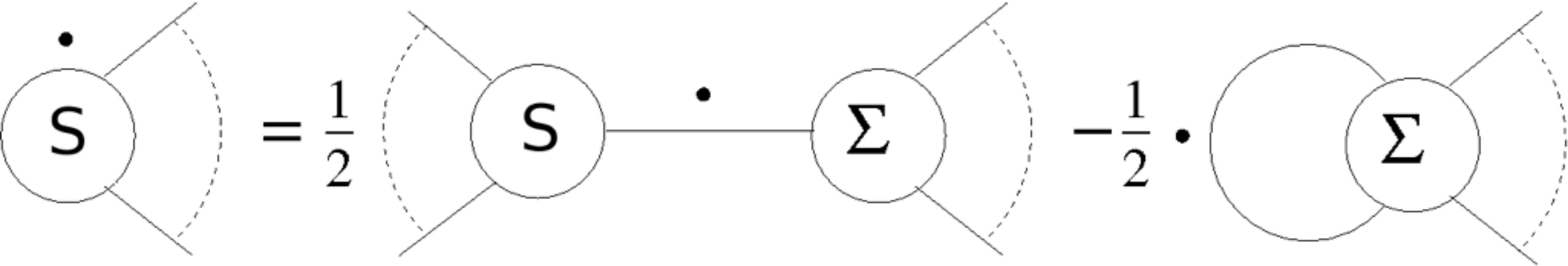
K.G. Wilson & J. Kogut, Phys. Rep. **12C** (1974) 75;
J Polchinski, Nucl. Phys. B231 (1984) 269

$$\Psi(x) = \frac{1}{2} \int_y \dot{\Delta}(x,y) \frac{\delta \Sigma}{\delta \varphi(y)} \qquad \Delta = c(p^2/\Lambda^2)/p^2$$



$$\Sigma = S - 2\hat{S} \qquad \hat{S} = \frac{1}{2} \partial_\mu \varphi \cdot c^{-1} \cdot \partial_\mu \varphi$$

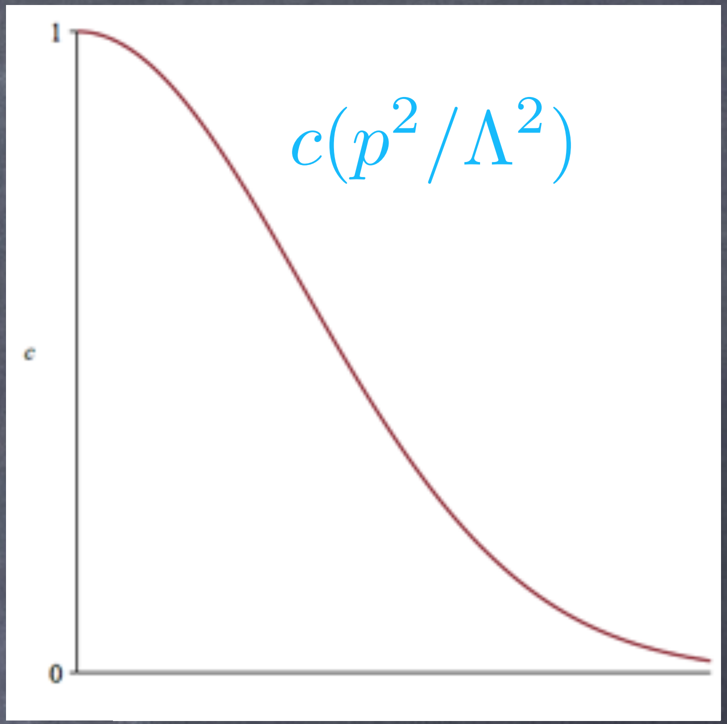
$$\dot{S} = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$



$$\dot{S} = \int_x \Psi(x) \frac{\delta S}{\delta\varphi(x)} - \int_x \frac{\delta \Psi(x)}{\delta\varphi(x)}$$

Generalised scalar flow eqn

S. Arnone, A. Gatti & TRM, Exact scheme dependence at one loop, JHEP 05 (2002) 059; + O.J. Rosten, ... at two loops, Phys. Rev. D69 (2004) 065009



$$\Psi(x) = \frac{1}{2}\int_y \dot{\Delta}(x,y)\frac{\delta\Sigma}{\delta\varphi(y)} \qquad \Delta = c(p^2/\Lambda^2)/p^2$$

$$\Sigma = S - 2\hat{S} \qquad \hat{S} = \frac{1}{2}\partial_\mu\varphi \cdot c^{-1} \cdot \partial_\mu\varphi + \cdots$$

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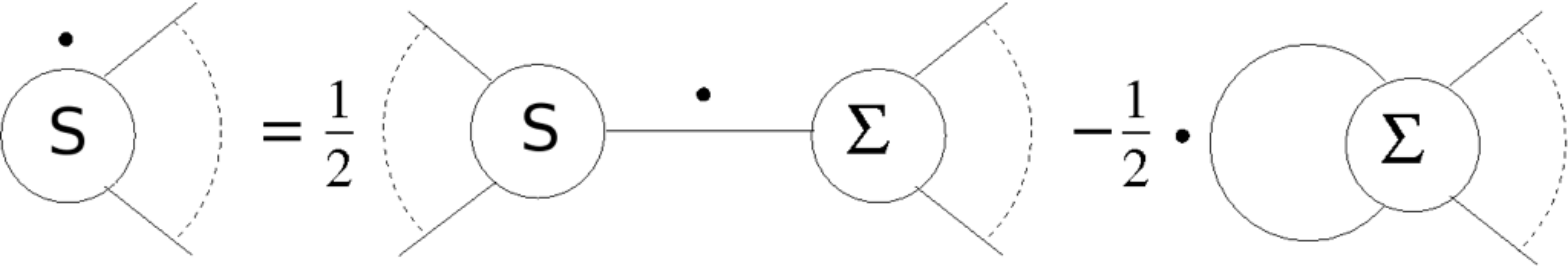


Requiring at classical level, $S^{\varphi\varphi} = \hat{S}^{\varphi\varphi} = p^2/c$

$$\dot{S}^{\varphi\varphi} = -S^{\varphi\varphi}\dot{\Delta}S^{\varphi\varphi}$$

agrees with

$$\Delta = \left(S^{\varphi\varphi}\right)^{-1}$$

Use this later to determine kernel

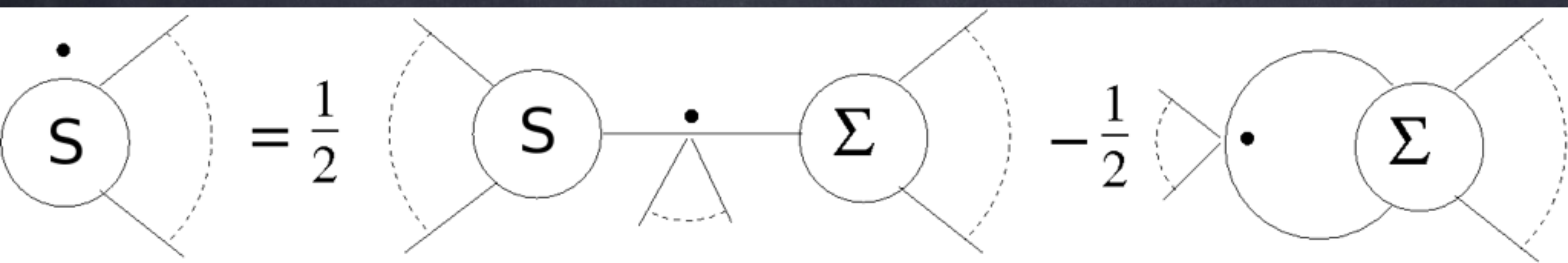
non-Abelian gauge theory

$$\dot{S} = \frac{1}{2}\frac{\delta S}{\delta A_\mu}\cdot\{\dot{\Delta}\}\cdot\frac{\delta \Sigma_g}{\delta A_\mu} - \frac{1}{2}\frac{\delta}{\delta A_\mu}\cdot\{\dot{\Delta}\}\cdot\frac{\delta \Sigma_g}{\delta A_\mu}$$

Some covariantization e.g. $\dot{\Delta}(-\partial^2) \mapsto \dot{\Delta}(-D^2)$

- Gauge invariant flow equation
- Can solve the equation **without fixing the gauge**!
- SU(N) YM, QCD, QED, one loop & two-loop β functions

TRM + S. Arnone, Yu. A. Kubyshin, J.F. Tighe, A. Gatti, O. J. Rosten: 29 papers from 1995 - 2011



non-Abelian gauge theory

$$D_\mu := \partial_\mu - iA_\mu \qquad\qquad \delta A_\mu = [D_\mu, \omega(x)]$$

$$A^R_\mu = Z^{-1/2} A_\mu \quad \Longrightarrow \quad \delta A^R_\mu = Z^{-1/2}\partial_\mu \omega - i[A^R_\mu, \omega]$$

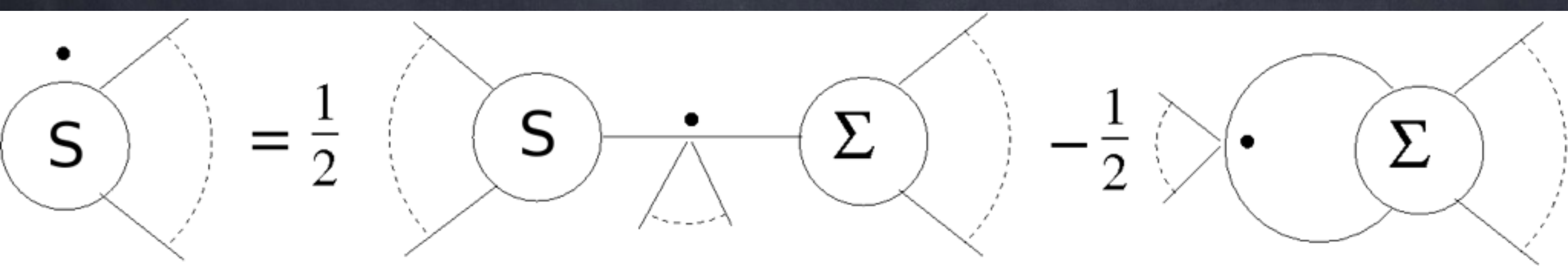
No wavefunction renormalisation.

Only g renormalises

non-Abelian gauge theory

$$\dot{S} = \frac{1}{2}\frac{\delta S}{\delta A_\mu}\cdot\{\dot{\Delta}\}\cdot\frac{\delta \Sigma_g}{\delta A_\mu} - \frac{1}{2}\frac{\delta}{\delta A_\mu}\cdot\{\dot{\Delta}\}\cdot\frac{\delta \Sigma_g}{\delta A_\mu}$$

$$S[A](g) = \frac{1}{g^2}\underbrace{\frac{1}{4}\mathrm{tr}\int_x F_{\mu\nu}\, c^{-1}\left(-\frac{D^2}{\Lambda^2}\right)F_{\mu\nu}}_{\hat{S}} + \mathcal{O}(F^3) + \cdots$$



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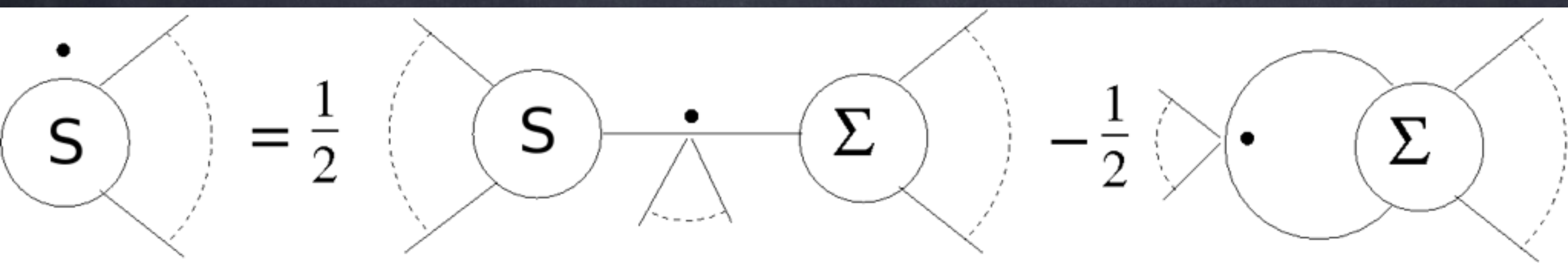
$$\Sigma_g = g^2 S - 2\hat{S}$$



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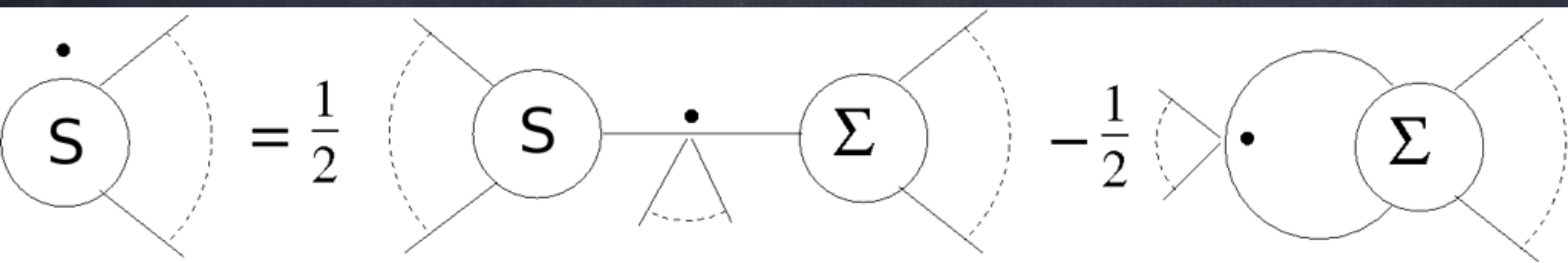
$$\Sigma_g = g^2 S - 2\hat{S} \qquad\qquad S = \frac{1}{g^2}S_0 + S_1 + g^2 S_2 + \cdots$$



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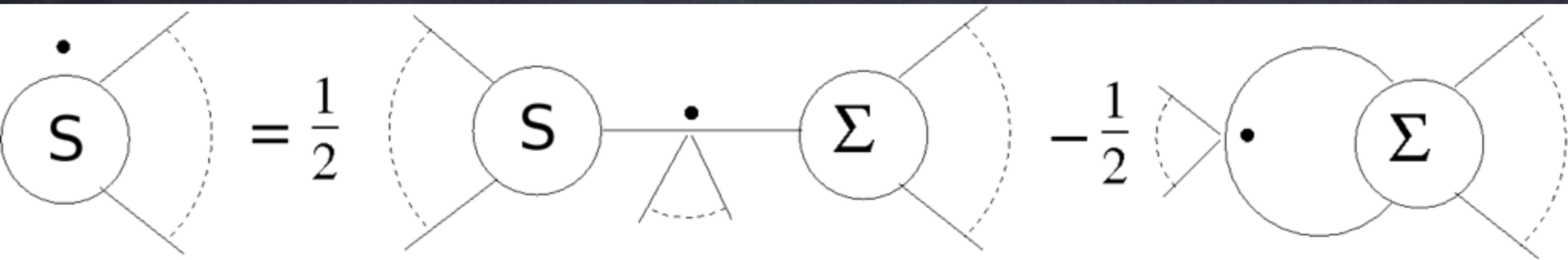
$$\Sigma_g = g^2 S - 2\hat{S} \qquad\qquad S = \frac{1}{g^2}S_0 + S_1 + g^2 S_2 + \cdots$$

$$\beta := \dot{g} = \beta_1 g^3 + \beta_2 g^5 + \cdots$$

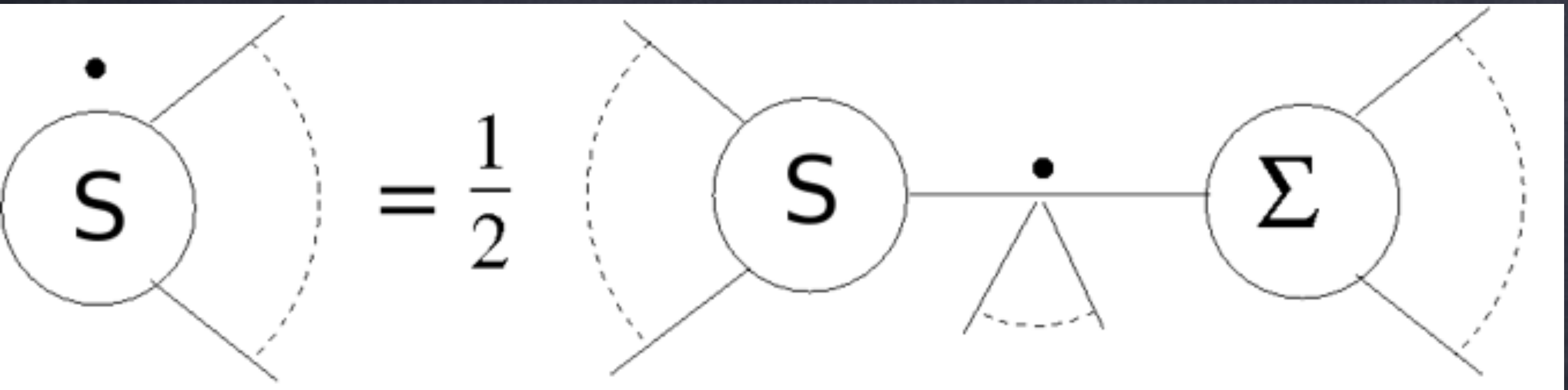


classical limit: g->0

$$\dot{S} = \frac{1}{2}\frac{\delta S}{\delta A_\mu} \cdot \{\dot{\Delta}\} \cdot \frac{\delta \Sigma}{\delta A_\mu}$$

$$\Sigma \equiv \Sigma_0 = S_0 - 2\hat{S}$$

$$S = \frac{1}{g^2} S_0$$



classical limit: g->0

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$$S^{AA}_{\mu\nu} = (\delta_{\mu\nu}p^2 - p_\mu p_\nu)\, c^{-1}(p^2/\Lambda^2)$$

$$\dot{S}^{AA}_{\mu\nu} = -S^{AA}_{\mu\alpha}\dot{\Delta}S^{AA}_{\alpha\nu}$$

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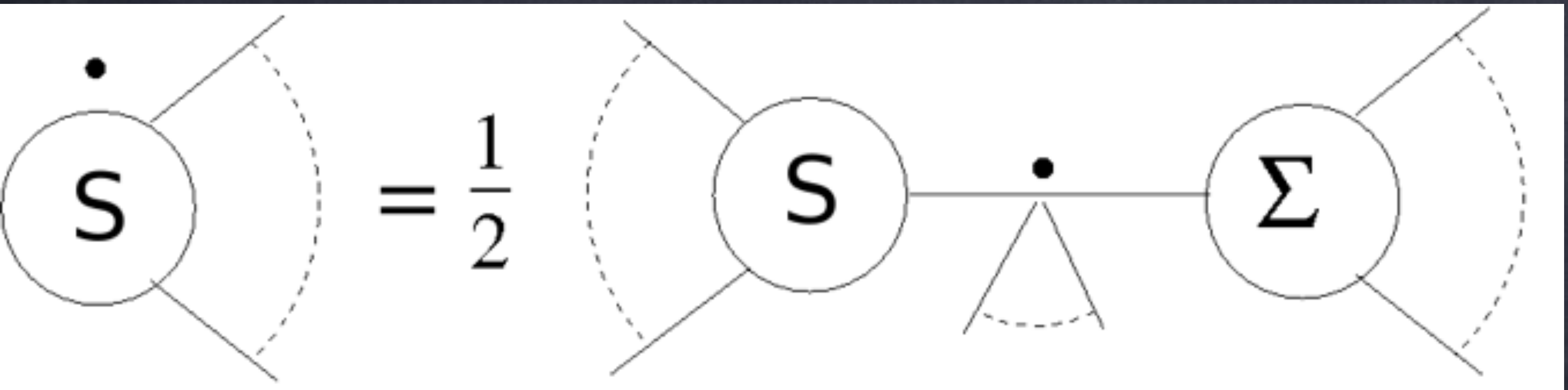

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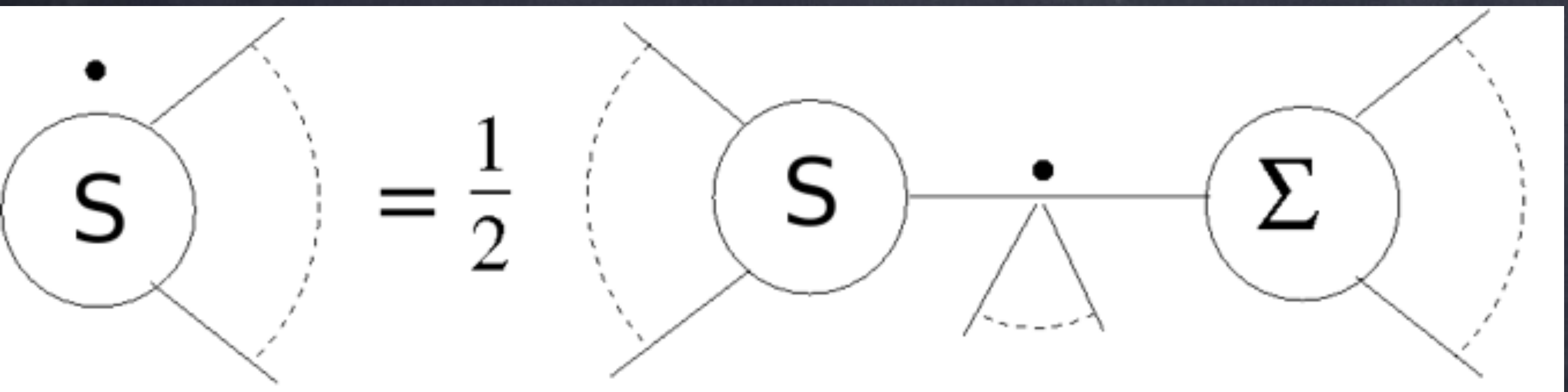
$$\dot{S}^{AA}_{\mu\nu} = -S^{AA}_{\mu\alpha}\dot{\Delta}S^{AA}_{\alpha\nu}$$

$$\Delta = c/p^2$$ **but** $$\Delta S^{AA}_{\mu\nu} = \delta_{\mu\nu} - p_\mu p_\nu/p^2$$

$$\Sigma \equiv \Sigma_0 = S_0 - 2\hat{S}$$

$$S = \frac{1}{g^2}S_0$$



Regularisation

Covariant higher derivatives are not enough

$c^{-1} \sim D^{2(n-1)}/\Lambda^{2(n-1)}$ but $S_1 \sim \mathrm{Tr} \ln D^{2n} = n \, \mathrm{Tr} \ln D^2$

Embed in spontaneously broken supergroup

Degrees of freedom cancel at high energies

$$A_\mu = \begin{pmatrix} A^1_\mu & B_\mu \\ \bar{B}_\mu & A^2_\mu \end{pmatrix}$$

(Parisi-Sourlas SUSY)

Massive ~ Λ fermionic

gauge-invariant

Pauli-Villars

Gravity

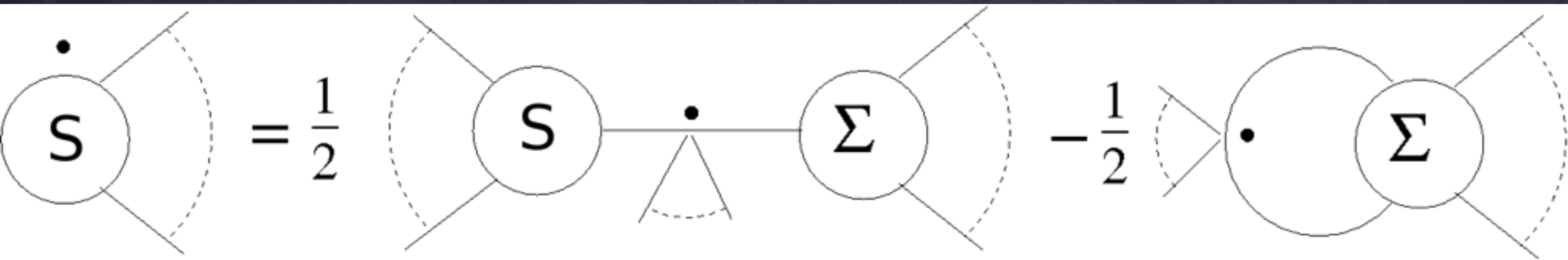
$$\dot{S} = \frac{1}{2}\frac{\delta S}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}} - \frac{1}{2}\frac{\delta}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}}$$

$$K_{\mu\nu\rho\sigma}(x,y) = \frac{1}{\sqrt{g}}\delta(x-y)\left(g_{\mu(\rho}g_{\sigma)\nu} + j g_{\mu\nu}g_{\rho\sigma}\right)\dot{\Delta}(-\nabla^2)$$

DeWitt parameter

$j \to \infty$ conformal truncation

$j = -1/4$ unimodular



Gravity

$$\dot{S} = -a_0[S, \Sigma] + a_1[\Sigma]$$

$$\dot{S} = \frac{1}{2}\frac{\delta S}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}} - \frac{1}{2}\frac{\delta}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}}$$

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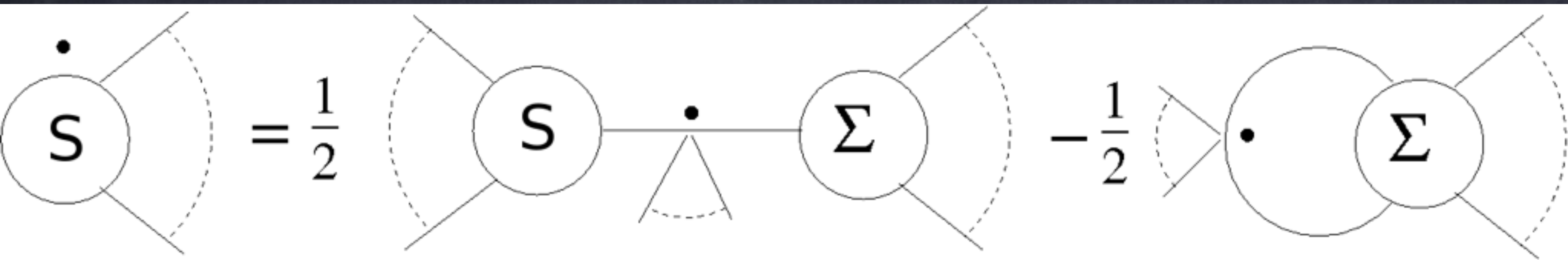

Gravity

$$\dot{S} = -a_0[S, \Sigma] + a_1[\Sigma]$$

$$\tilde{\kappa}(\Lambda) = 32\pi G = 4/M^2$$

$$\Sigma = \tilde{\kappa} S - 2\hat{S}$$

$$S = \frac{1}{\tilde{\kappa}} S_0 + S_1 + \tilde{\kappa} S_2 + \tilde{\kappa}^2 S_3 + \cdots$$

Gravity

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$$\tilde{\kappa}(\Lambda) = 32\pi G = 4/M^2 \qquad \Sigma = \tilde{\kappa} S - 2\hat{S}$$

$$S = \frac{1}{\tilde{\kappa}} S_0 + S_1 + \tilde{\kappa} S_2 + \tilde{\kappa}^2 S_3 + \cdots$$

$$\frac{1}{\tilde{\kappa}}\dot{S}_0 + \dot{S}_1 + \tilde{\kappa}\dot{S}_2 + \tilde{\kappa}^2\dot{S}_3 + \cdots + \beta\left(-\frac{1}{\tilde{\kappa}^2}S_0 + S_2 + 2\tilde{\kappa}S_3 + \cdots\right) = -\frac{1}{\tilde{\kappa}}a_0[S_0, S_0 - 2\hat{S}]$$

$$-2a_0[S_0 - \hat{S}, S_1] + a_1[S_0 - 2\hat{S}] + \tilde{\kappa}\left(-2a_0[S_0 - \hat{S}, S_2] - a_0[S_1, S_1] + a_1[S_1]\right) + \cdots$$

$$\beta := \dot{\tilde{\kappa}} = \beta_1 \Lambda^2 \tilde{\kappa}^2 + \beta_2 \Lambda^4 \tilde{\kappa}^3 + \cdots$$

Gravity

$$\dot{S} = -a_0[S, \Sigma] + a_1[\Sigma]$$

$$\tilde{\kappa}(\Lambda) = 32\pi G = 4/M^2 \qquad \Sigma = \tilde{\kappa} S - 2\hat{S}$$

$$S = \frac{1}{\tilde{\kappa}} S_0 + S_1 + \tilde{\kappa} S_2 + \tilde{\kappa}^2 S_3 + \cdots$$

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$$\beta := \dot{\tilde{\kappa}} = \beta_1 \Lambda^2 \tilde{\kappa}^2 + \beta_2 \Lambda^4 \tilde{\kappa}^3 + \cdots$$

$$\kappa(\Lambda) = \tilde{\kappa}\Lambda^2 = 4\Lambda^2/M^2$$

$$\beta(\kappa) = \dot{\kappa} = 2\kappa + \beta_1 \kappa^2 + \beta_2 \kappa^3 + \cdots$$

classical Gravity

$$\dot{S} = -a_0[S, \Sigma]$$

Actions have dimn -2

$$\Sigma = S - 2\hat{S}$$

classical Gravity

$$\dot{S} = -a_0[S, \Sigma]$$

Actions have dimn -2

$$\Sigma = S - 2\hat{S}$$

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$$

$$S = \int đp\, \delta(p) \mathcal{S}^{\mu\nu}(p) h_{\mu\nu}(p) + \frac{1}{2} \int đp\, đq\, \delta(p+q) \mathcal{S}^{\mu\nu\rho\sigma}(p,q) h_{\mu\nu}(p) h_{\rho\sigma}(q)$$

$$+ \frac{1}{3!} \int đp\, đq\, đr\, \delta(p+q+r) \mathcal{S}^{\mu\nu\rho\sigma\alpha\beta}(p,q,r) h_{\mu\nu}(p) h_{\rho\sigma}(q) h_{\alpha\beta}(r) + \cdots$$

classical Gravity

$$\dot{S} = -a_0[S, \Sigma]$$

Actions have dimn -2

$$\Sigma = S - 2\hat{S}$$

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$$

$$S = \int đp\, \delta(p) \mathcal{S}^{\mu\nu}(p) h_{\mu\nu}(p) + \frac{1}{2} \int đp\, đq\, \delta(p+q) \mathcal{S}^{\mu\nu\rho\sigma}(p,q) h_{\mu\nu}(p) h_{\rho\sigma}(q)$$
$$+ \frac{1}{3!} \int đp\, đq\, đr\, \delta(p+q+r) \mathcal{S}^{\mu\nu\rho\sigma\alpha\beta}(p,q,r) h_{\mu\nu}(p) h_{\rho\sigma}(q) h_{\alpha\beta}(r) + \cdots$$

Diffeomorphism invariance is exact!

$$-2p_{1\mu_1} \mathcal{S}^{\mu_1\nu_1\cdots\mu_n\nu_n}(p_1, \cdots, p_n) = \sum_{i=2}^{n} \pi_{2i} \Big\{ p_2^{\nu_1} \mathcal{S}^{\mu_2\nu_2\cdots\mu_n\nu_n}(p_1+p_2, p_3, \cdots, p_n)$$
$$+2p_{1\alpha} \delta^{\nu_1(\nu_2} \mathcal{S}^{\mu_2)\alpha\mu_3\nu_3\cdots\mu_n\nu_n}(p_1+p_2, p_3, \cdots, p_n) \Big\}$$

classical Gravity

Two-point vertex:

$$\frac{1}{2}\left(h_{\mu\nu}p^2h^{\mu\nu} - hp^2h + 2h^{\mu\nu}p_\mu p_\nu h - 2h^{\mu\nu}p_\mu p_\rho h_\nu{}^\rho\right) + O(p^4)$$

Renormalisation condition:

$$S = \int d^4x\,\sqrt{g}\,\{g_0 + g_2(-2R) + g_{4,1}R^2 + g_{4,2}R^{\mu\nu}R_{\mu\nu} + g_{6,1}R^3 + g_{6,2}RR^{\mu\nu}R_{\mu\nu} + \cdots$$

$g_2 \leftarrow 1$

classical Gravity

Two-point vertex:

$$\frac{1}{2}\left(h_{\mu\nu}p^2h^{\mu\nu} - hp^2h + 2h^{\mu\nu}p_\mu p_\nu h - 2h^{\mu\nu}p_\mu p_\rho h_\nu{}^\rho\right)c^{-1}(p^2/\Lambda^2)$$

$$\dot{S} = \frac{1}{2}\frac{\delta S}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}} - \frac{1}{2}\frac{\delta}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}}$$

$$K_{\mu\nu\rho\sigma}(x,y) = \frac{1}{\sqrt{g}}\delta(x-y)\left(g_{\mu(\rho}g_{\sigma)\nu} + j g_{\mu\nu}g_{\rho\sigma}\right)\dot{\Delta}(-\nabla^2)$$

$$j = -1/2$$

$$\Delta = c/p^2$$

classical Gravity

Two-point vertex:

$$\frac{1}{2}\left(h_{\mu\nu}p^2h^{\mu\nu} - hp^2h + 2h^{\mu\nu}p_\mu p_\nu h - 2h^{\mu\nu}p_\mu p_\rho h_\nu{}^\rho\right) c^{-1}(p^2/\Lambda^2)$$

$$c^{-1} = 1 + \frac{p^2}{\Lambda^2}\, d\left(\frac{p^2}{\Lambda^2}\right)$$

$$\hat{\mathcal{L}} = -2R + \frac{2}{\Lambda^2}R_{\mu\nu}\, d(-\nabla^2/\Lambda^2)R^{\mu\nu} - \frac{1}{\Lambda^2}R\, d(-\nabla^2/\Lambda^2)R$$

$$\mathcal{L} = \hat{\mathcal{L}} + O(R^3)$$

classical Gravity

$$\dot{\mathcal{L}} = -\mathsf{a}_0[\mathcal{L}, \mathcal{L} - 2\hat{\mathcal{L}}]$$

$$\mathcal{L} = \sum_{i=0}^{\infty} \sum_{\alpha_i} g_{2i,\alpha_i} \, \mathcal{O}_{2i,\alpha_i}$$

$$\dot{S} = \frac{1}{2} \frac{\delta S}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}} - \frac{1}{2} \frac{\delta}{\delta g_{\mu\nu}} \cdot K_{\mu\nu\rho\sigma} \cdot \frac{\delta \Sigma}{\delta g_{\rho\sigma}}$$

$$K_{\mu\nu\rho\sigma}(x,y) = \frac{1}{\sqrt{g}} \delta(x-y) \left(g_{\mu(\rho} g_{\sigma)\nu} + \jmath g_{\mu\nu} g_{\rho\sigma} \right) \dot{\Delta}(-\nabla^2)$$

$$\dot{\Delta}(-\nabla^2) = \sum_{k=0}^{\infty} \frac{1}{k!} \dot{\Delta}^{(k)}(0) \left(-\nabla^2 \right)^k$$

Background independent calculations!

classical Gravity

$$\dot{\mathcal{L}} = -\mathsf{a}_0[\mathcal{L}, \mathcal{L} - 2\hat{\mathcal{L}}]$$

$$\mathcal{L} = \sum_{i=0}^{\infty}\sum_{\alpha_i} g_{2i,\alpha_i}\,\mathcal{O}_{2i,\alpha_i}$$

$$\{g_0 + g_2(-2R) + g_{4,1}R^2 + g_{4,2}R^{\mu\nu}R_{\mu\nu} + g_{6,1}R^3 + g_{6,2}RR^{\mu\nu}R_{\mu\nu} + \cdots\}$$

$$\mathsf{a}_0[\mathcal{O}_d, 1] = -\frac{1}{8}(d-4)\dot{\Delta}(0)\,\mathcal{O}_d$$

$$\dot{g}_2 = -\frac{c'(0)}{\Lambda^2}\hat{g}_0 \qquad \Longrightarrow \qquad \hat{g}_0 = 0$$

$$\dot{g}_0 = \frac{c'(0)}{\Lambda^2}g_0^2$$

classical Gravity

$$\dot{g}_{4,1} R^2 + \dot{g}_{4,2} R^{\mu\nu} R_{\mu\nu} = -2 \frac{c'(0)}{\Lambda^2} \left(R^2 - 2 R_{\mu\nu} R^{\mu\nu} \right)$$

classical Gravity

$$\dot{g}_{4,1}R^2 + \dot{g}_{4,2}R^{\mu\nu}R_{\mu\nu} = -2\frac{c'(0)}{\Lambda^2}\left(R^2 - 2R_{\mu\nu}R^{\mu\nu}\right)$$

$g_{4,\alpha} \propto 1/\Lambda^2$ at fixed point

$$\Longrightarrow \; g_{4,1} = c'(0)/\Lambda^2\,, \quad g_{4,2} = -2c'(0)/\Lambda^2$$

classical Gravity

$$\dot{g}_{4,1} R^2 + \dot{g}_{4,2} R^{\mu\nu} R_{\mu\nu} = -2 \frac{c'(0)}{\Lambda^2} \left(R^2 - 2 R_{\mu\nu} R^{\mu\nu} \right)$$

$g_{4,\alpha} \propto 1/\Lambda^2$ at fixed point

$$\Longrightarrow \quad g_{4,1} = c'(0)/\Lambda^2 \,, \qquad g_{4,2} = -2c'(0)/\Lambda^2$$

$$c^{-1} = 1 + \frac{p^2}{\Lambda^2}\, d\left(\frac{p^2}{\Lambda^2}\right)$$

$$\hat{\mathcal{L}} = -2R + \frac{2}{\Lambda^2} R_{\mu\nu}\, d(-\nabla^2/\Lambda^2) R^{\mu\nu} - \frac{1}{\Lambda^2} R\, d(-\nabla^2/\Lambda^2) R$$

$$\mathcal{L} = \hat{\mathcal{L}} + O(R^3)$$

consistency

Gravity

Covariant higher derivatives are not enough

Extend diffeomorphisms along fermionic directions?

$$x^A = (x^\mu, \theta^a)$$

$$ds^2 = dx^A g_{AB} dx^B$$

Degrees of freedom cancel at high energies
(Parisi-Sourlas SUSY)

$$10\ g_{\mu\nu} + 6\ g_{ab} - 16\ (g_{\mu a} = -g_{a\mu})$$

+ spontaneous symmetry breaking ...

Manifestly diffeomorphism invariant exact RG

Diffeomorphism invariant calculations

Background independent calculations

Manifestly diffeomorphism invariant classical exact RG

Diffeomorphism invariant calculations

Background independent calculations