Black hole engineering in short scale modified gravity

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2nd FLAG Meeting, Trento 6-8 June 2016

Talk partially based on

PN & E. Spallucci, Adv. High En. Phys. **2014** (2014) 805684

A.M. Frassino, S. Köppel, PN, Entropy **18** (2016) 181

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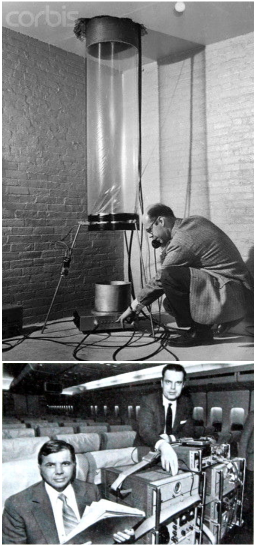
Gravity $\longrightarrow$ General Relativity

- Classical tests of GR
  1. the perihelion precession of Mercury's orbit
  2. the deflection of light by the Sun
  3. the gravitational redshift of light
- Modern tests of GR
  1. Hulse-Taylor binary
  2. direct dection of GWs
  3. ...
- Most accurate theory in physics
- Weakly constrained at short scales



Pound-Rebka (1959), Hafele-Keating (1971)

Short scale limits

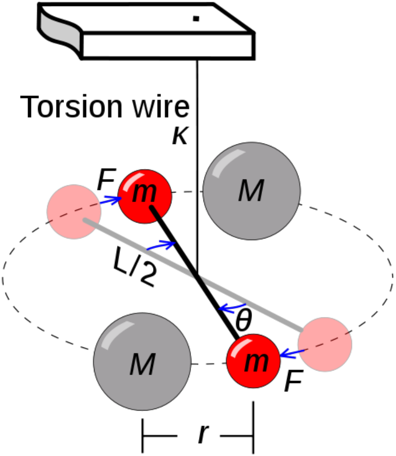
- deviations of Newton's law

$$\begin{aligned}\Phi &= -G\frac{M}{r}\left(1+\alpha e^{-r/R_*}\right)\\ &\Rightarrow \quad R_* < 44\ \mu\mathrm{m}\end{aligned}$$

- $f(R)$-gravity tests

$$\begin{aligned}f(R) &\simeq a_0 + a_1 R + a_2 R^2 + \dots\\ &\Rightarrow \quad \sqrt{a_2} < 63\ \mu\mathrm{m}\end{aligned}$$

[Cembranos (2009), Berry and Gair (2011), Capozziello & De Laurentis (2011)]



Scale drawing of the torsion pendulum and rotating attractor. See Hoyle *et al.*, PRL **86**, 1418 (2001).

Theoretical issues of GR⟶Necessity of quantum gravity

Point of view

Any approach towards quantum structure of spacetime is useful iff it has something sensible to say about:

- Singularity in cosmology
- Black hole singularity
- Cosmological constant problem

– T. Padmanabahn, 1st Karl Schwarzschild Meeting, Frankfurt am Main, 2013

I would add at least...

- Microscopic origin of black hole thermodynamics
- Information paradox
- (Hierarchy problem)

Top-Down approach

Formulate a theory → get phenomenological predictions

Difficulties...

- Theories are hard to formulate
- Several competing theories on market
- Few phenomenological predictions

For black holes

- String theory [Strominger & Vafa, 1996; Mathur, 2005]
- Loop Quantum Gravity [Modesto, 2006; Rovelli & Vidotto, 2014]
- Asymptotically safe gravity [Bonanno & Reuter, 2000]
- Effective theories
  - Noncommutative geometry [Calmet & Kobakhidze, 2005; PN, Smailagic & Spallucci, 2006]
  - Generalized uncertainty principle [Maggiore, 1993; Alder, Chen & Santiago, 2001; Carr, Mureika & PN, 2015]
  - Nonlocal gravity [Modesto, Moffat & PN, 2011]
  - Strong electrodynamics [Bardeen, 1968; Beato & Garcia, 1998]
  - Unparticles [Gaete, Helayel-Neto, Spallucci, 2010; Mureika & Spallucci, 2010]

Bottom-up approach

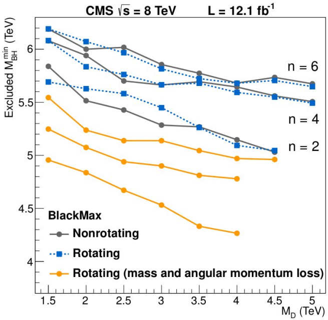
Get experimental data $\rightarrow$ reconstruct the theory that fits them

Difficulties...

- absence/paucity of data
- narrow observational window around M_{P}

Terascale black holes signatures

- Hadronic/leptonic activity $\sim$ 5:1
- Visible transverse energy of order 30% of the total energy
- Suppression of hard di-jets events
- Emission of a few hard visible quanta
- Black hole remnants
  - neutral: modified momentum distribution
  - charged: ionizing tracks



- rules out BHs with $M = 4.5 - 6.5\ \mathrm{TeV}$.
- **CAVEAT:** semiclassical analysis breakdown!

Spacetime engineering

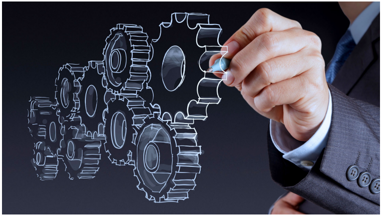
Assume a Principle $\rightarrow$ derive/postulate a consistent spacetime $\rightarrow$ derive phenomenology $\rightarrow$ falsify/confirm the Principle

Difficulties...

- consistency of the proposed spacetime
- paucity/absence of data

Advantages...

- viable derivation (toy model)
- exposure of new effects



Examples of black hole engineering

Principle: curvature finiteness

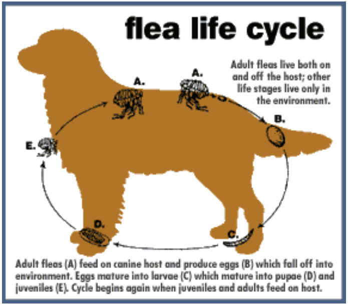
- Dymnikova's vacuum non-singular black holes, GERG 24, 235 (1992)
- Hayward's regular black hole, PRL 96, 031103 (2006)
- Manasse-Kazanas's regular black holes, PRD 72, 024016 (2005)

Planckian black holes

- Planckian graviton scattering $\to$ bound state i.e. black hole
- Mass $\sim M_{\mathrm{P}} \sim 10^{19}\,\mathrm{GeV} = 2 \times 10^{-8}\,\mathrm{kg}$, Size $\sim L_{\mathrm{P}} \sim 1.6 \times 10^{-35}$ m
- Fundamental qubit

$$N \sim \frac{r_+^2}{L_{\mathrm{P}}^2}$$

- Classical metrics for $N \to \infty$
- Spacetime dissolution at the Planck scale



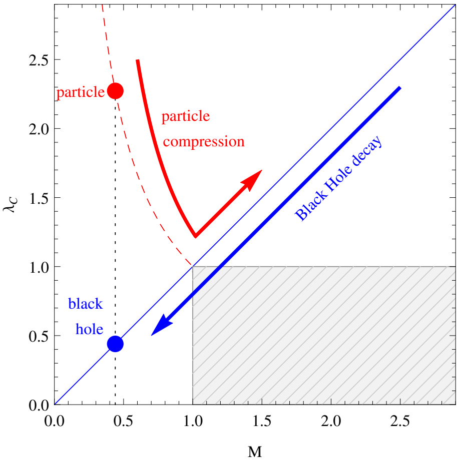
The Planck scale

- Quantum N-portrait i.e. black hole as a condensate of N gravitons [Dvali and Gomez, 2013; ...]
- Horizon wave function formalism [Casadio, 2013; ...]
- Particle-like, non-geometric black hole description [Smailagic and Spallucci, 2016]
- Black hole precursors [Calmet, 2014]

Gravity ultraviolet self-completeness

- Planck scale separates phases: “Particles” and ”BHs”
- Neo-classical regime for $M \gg M_{\rm P}$
- Singularity is masked – not accessible
- Spacetime at scales $\leq L_{\rm P}$ is dissolved.
- GUP interpolates the two regimes

$$R \sim \frac{1}{M} + L_{\rm P}^2 M$$



ASDGFGC phase diagram
[Aurilia & Spallucci, 2003]
[Dvali & Gomez, 2010]
[Dvali, Folkerts & Germani, 2010]
[Carr, 2013]

Problem

- The Schwarzschild metric does not fit in the scheme!

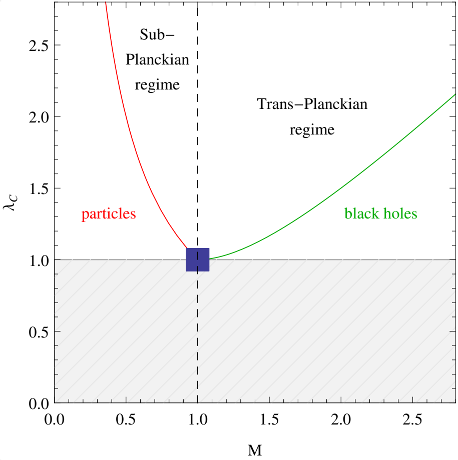
Recipe for black hole engineering

List of ingredients

- Inaccessibility of length scales below L_{P}
- "Particle-BH" phase separation
- Asymptotic state at L_{P} and M_{P} = quibit (basic information capacity)
- Universal parameter N
    - $r_{\mathrm{h}} = \sqrt{N} L_{\mathrm{P}}$
    - $M = \sqrt{N} M_{\mathrm{P}}$
- Schwarzschild limit for $N \to \infty$

Cooking methods

1. Turning on an effective $\mathfrak{T}_{\mu\nu}$
2. Gravity action modifications $S = \frac{c^4}{16\pi G}\int \mathcal{R}\sqrt{-g}\,\mathrm{d}^4x \;\to S = \frac{c^4}{16\pi G}\int f(\mathcal{R}, \Box, \dots)\sqrt{-g}\,\mathrm{d}^4x$,



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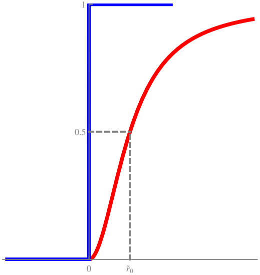
General set up

- Particle energy density

$$\rho_{\mathrm{p}} = \frac{M}{4\pi r^2}\delta(r) = \frac{M}{4\pi r^2}\frac{\mathrm{d}\Theta(r)}{\mathrm{d}r}$$

- Planck scale corrections $\Theta(r) \to h(r)$

$$\rightsquigarrow \mathfrak{T}_{\mu}^{\ \nu} = \mathrm{diag}\left(-\rho,\ p_r,\ p_\perp,\ p_\perp\right)$$



$$\mathrm{d}s^2 = e^{2\Phi(r)}\left(1 - 2Gm(r)/r\right)\mathrm{d}t^2 - \left(1 - 2Gm(r)/r\right)^{-1}\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega^2$$

- $m(r) = 4\pi\int \mathrm{d}r\ r^2\ \rho(r) = M\int \mathrm{d}r\ \frac{\mathrm{d}h(r)}{\mathrm{d}r} \to m(r) = M\ h(r)$

$$\nabla_\mu \mathfrak{T}^{\mu\nu} = 0 \rightsquigarrow \frac{\mathrm{d}p_r}{\mathrm{d}r} = -G\,\frac{m(r) + 4\pi\ r^3 p_r}{r(r - 2Gm(r))}\left(\rho + p_r\right) + \frac{2}{r}\left(p_\perp - p_r\right)$$

- Equation of state

$$p_r = -\rho \Leftrightarrow \Phi(r) = 0$$

$\rightsquigarrow$“clean” black hole solution

Requirements→metric derivation

Recap

Singularity inaccessibility
Particle/BH separation
Planckian asymptotic state
+
Only one "natural scale"

$\rightsquigarrow$ **Planckian extremal configuration!**

- two horizons $r_\pm$ coalescing in $r_\mathrm{e} = r_+ = r_-$.

$$r_\mathrm{e} \stackrel{!}{=} \sqrt{G} = L_\mathrm{P}$$

$$M_\mathrm{e} \stackrel{!}{=} 1/\sqrt{G} = M_\mathrm{P}$$

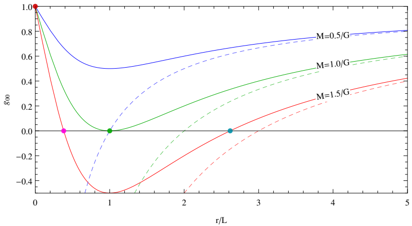
- solution ("Holographic metric") [PN and Spallucci, 2014]

$$\rightsquigarrow \mathrm{d}s^2 = -\left(1 - \frac{2L_\mathrm{P}^2\, M\, r}{r^2 + L_\mathrm{P}^2}\right)\mathrm{d}t^2 + \left(1 - \frac{2L_\mathrm{P}^2\, M\, r}{r^2 + L_\mathrm{P}^2}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Holographic metric – horizons and else

- Event horizon

$$r_+(N) = L_{\mathrm{P}}^2 \left(M + \sqrt{M^2 - M_{\mathrm{P}}^2} \right)$$
$$= \sqrt{N} L_{\mathrm{P}}$$
$$\frac{\Delta r_+(N)}{r_+(N)} \sim \frac{1}{N^{3/2}}$$



$$M \gg M_{\mathrm{P}} \rightsquigarrow r_+ \simeq 2ML_{\mathrm{P}}^2 \left(1 - \frac{1}{N}\right)$$

$\rightsquigarrow$ **Macroquantumness!**, [Dvali and Gomez, 2012], [De Laurentis, PN & Rezzolla, in progress]

- Cauchy horizon

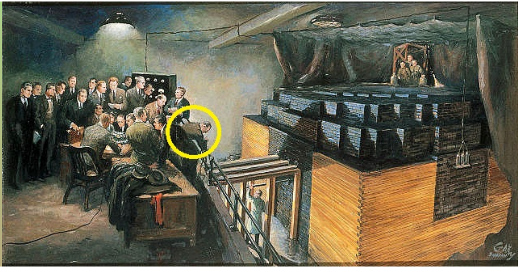
$$r_- = \frac{L_{\mathrm{P}}^2}{r_{\mathrm{h}}} = L_{\mathrm{P}}^2 \left(M - \sqrt{M^2 - M_{\mathrm{P}}^2} \right)$$
$$< L_{\mathrm{P}} \ (\text{not accessible!})$$

$\rightsquigarrow$ **no mass inflation!**

- "Light" objects, with $M < M_{\mathrm{P}}$, are (horizonless) "particles".

Holographic metric – Thermodynamics

SCRAM it off!



Safety Control Rod Axe Man (SCRAM) – Chicago Pile-1, Dec 2, 1942

Holographic metric – Thermodynamics

- Black hole SCRAM

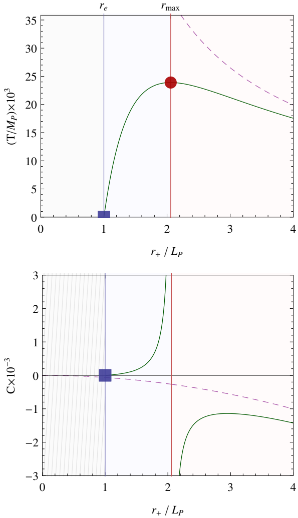
$$T = \frac{1}{4\pi r_+}\left(1 - \frac{2L_{\mathrm{P}}^2}{r_+^2 + L_{\mathrm{P}}^2}\right) = \frac{M_{\mathrm{P}}}{4\pi\sqrt{N}}\left(\frac{N-1}{N+1}\right)$$

- heat capacity

$$\mathcal{C} = -2\pi r_+\left(\frac{r_+^2 - L_{\mathrm{P}}^2}{L_{\mathrm{P}}^2}\right)\frac{(r_+^2 + L_{\mathrm{P}}^2)^2}{r_+^4 - 4L_{\mathrm{P}}^2 r_+^2 - L_{\mathrm{P}}^4}$$

- entropy

$$\begin{aligned} S(\mathcal{A}_+) &= \frac{\pi}{\mathcal{A}_0}\left(\mathcal{A}_+ - \mathcal{A}_0\right) + \pi \ln\left(\mathcal{A}_+/\mathcal{A}_0\right) \\ S(N) &= \pi\left(N + \ln(N) - 1\right) \end{aligned}$$



Black hole temperature and heat capacity.

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Holographic metric in AdS background

- AdS term disturbs the extremal configuration
- Metric Ansatz

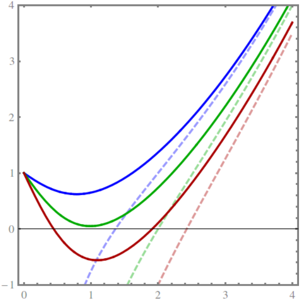
$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 + g_{rr}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$
$$-g_{00} = g_{rr}^{-1} = \left(1 - \frac{2L_\mathrm{P}^2\, M}{r}\, h_b(r) + \frac{r^2}{b^2}\right)$$

- Boundary conditions

$$h_b(r) \to 0 \quad \text{as} \quad r \to 0$$
$$h_b(r) \to 1 \quad \text{as} \quad r \to \infty$$
$$h_b(r) \to \frac{r^2}{r^2 + L_\mathrm{P}^2} \quad \text{as} \quad b \to \infty$$

- Planckian extremal black hole condition

$$r_\mathrm{e} \stackrel{!}{=} \sqrt{G} = L_\mathrm{P}$$
$$M_\mathrm{e} \stackrel{!}{=} 1/\sqrt{G} = M_\mathrm{P} + L_\mathrm{P}/b^2$$

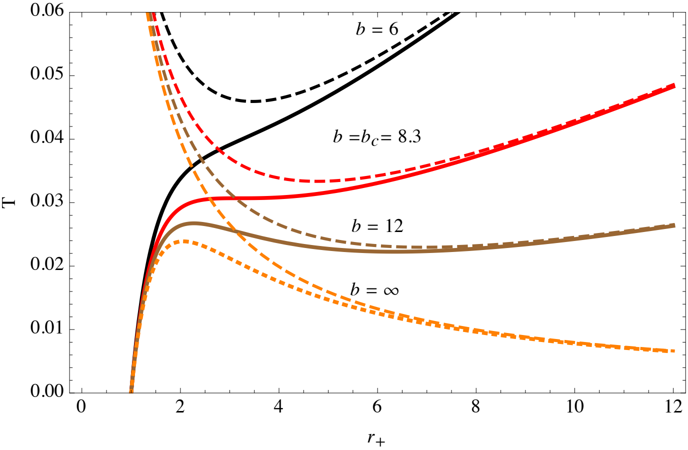


$$\rightsquigarrow h_b(r) = \frac{r^2}{r^2 + L_\mathrm{P}^2 + \frac{4L_\mathrm{P}^4}{b^2 - L_\mathrm{P}^2}}$$

Thermodynamics I

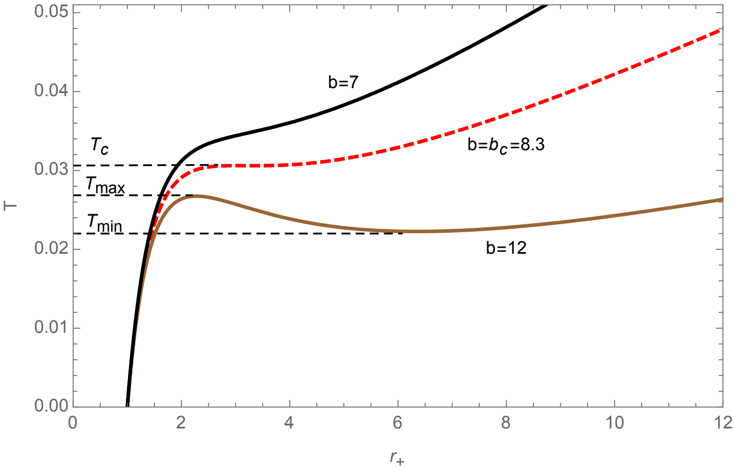
- Temperature

$$T = \frac{1}{4\pi r_+}\left[\left(1+\frac{r_+^2}{b^2}\right)\left(1 - r_+\frac{h_b'(r_+)}{h_b(r_+)}\right) + \frac{2r_+^2}{b^2}\right]$$



Note the critical value $b_c \simeq 8.3 L_{\mathrm{P}}$ where $T_{min} = T_{max} = T_c$.

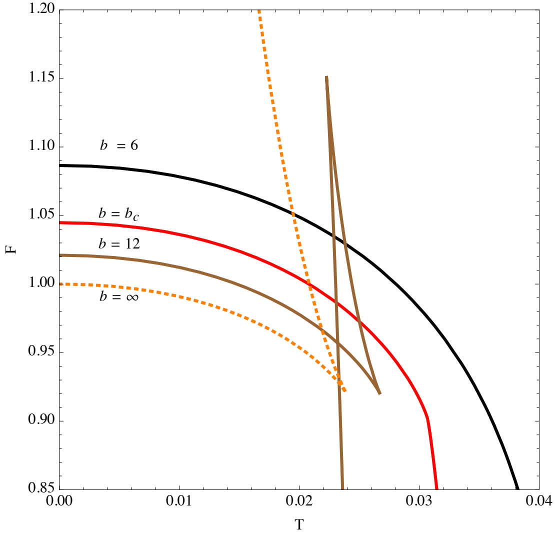
Thermodynamics II



Critical temperature

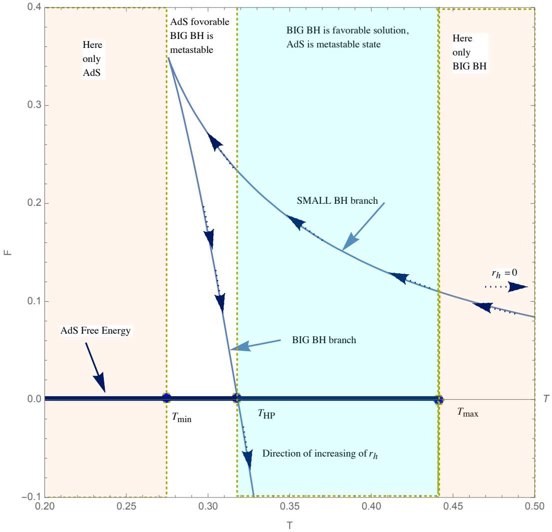
Thermodynamics III

- Free energy $F = F(T)$



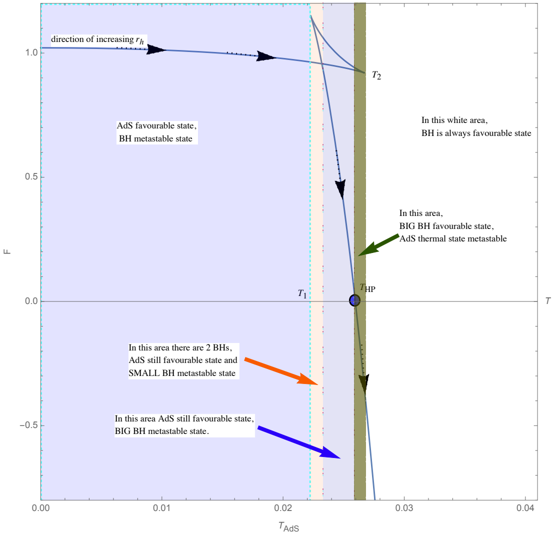
Thermodynamics IV

- Free energy $F = F(T)$ for Schwarzschild-AdS



Thermodynamics V

- Free energy $F = F(T)$ for Holographic-AdS



Thermodynamics VI

Summary

- $T < T_{min}$: One horizon, positive heat capacity, locally stable system but meta-stable state. System favours AdS-thermal background instead of BH;
- $T_{min} < T < T_{max}$: Three horizon radii $r_1 < r_2 < r_3$ possible, where r_1 and r_3 are stable but r_2 is locally unstable, so the mixed phase r_2 eventually decays to r_1 or r_3;
- $T = T_{tr}$: Phase transition between small and big black holes;
- $T_{tr} < T < T_{HP}$: AdS is still favorable and bigger black holes are meta-stable. The Hawking-Page temperature, defined by, $F(T_{HP}) = 0$, separates BHs and AdS as favored states;
- $T > T_{coll}$: AdS thermal radiation cannot longer sustain itself and collapses into black hole, as in regular Schwarzschild-AdS.

See also: Talk by Antonia Frassino this afternoon.

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Spacetime nucleation via instantons formalism

- Quantum state of the Universe [Gibbons and Hawking, 1973]

$$\Psi = \int D\left[g_{ab}\right] \; e^{-I_{\mathrm{E}}[g]}$$

- Semiclassically we have $I_{\mathrm{E}}[g] \approx I$

$$P = |\Psi|^2 \sim \exp\left(-2I\right).$$

- Two instantons:
  1. **background** I_{bg}
  2. **object** nucleated I_{obj} on the background
- the object/background pair nucleation rate is

$$\Gamma = \frac{P_{\mathrm{obj}}}{P_{\mathrm{bg}}} \sim \frac{\exp\left(-2I_{\mathrm{obj}}\right)}{\exp\left(-2I_{\mathrm{bg}}\right)}$$

- the background spacetime

$$ds_{\mathrm{E}}^2 = V(r)\, d\tau^2 + V(r)^{-1}\, dr^2 + r^2\, d\Omega^2$$

with $V(r) = 1 - \frac{\Lambda}{3} r^2$.

Production of scramming black holes

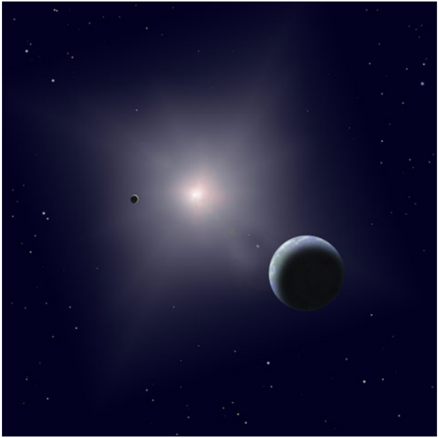
- for all istantons, rates are of the order [Mann and PN, 2011]

$$\Gamma \sim e^{-\frac{\pi}{\Lambda G}}$$

- however for the Cold Instanton

$$\Gamma_{\mathrm{C}} > \Gamma_{\mathrm{SdS}}$$

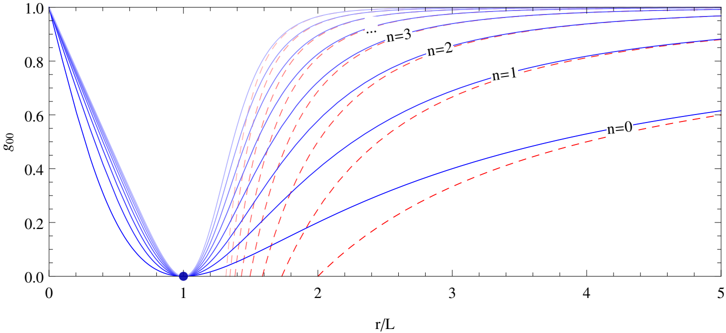
- *Nota bene*: for $\Lambda G \sim 1$ there is no I_{C}, but I_1 i.e. single horizon instanton.
- primordially produced BH scenario needs a revision!



Higher-dimensional holographic metric

- Consider the extension to $4 + n$ dimensions
- New scale M_*; $L_* = 1/M_*$
- Terascale extremal black holes

$$\rightsquigarrow h_n(r) = \frac{r^{2+n}}{r^{2+n} + L_*^{2+n}}.$$



Metric coefficient for $M = M_*$ in $4 + n$ dimension

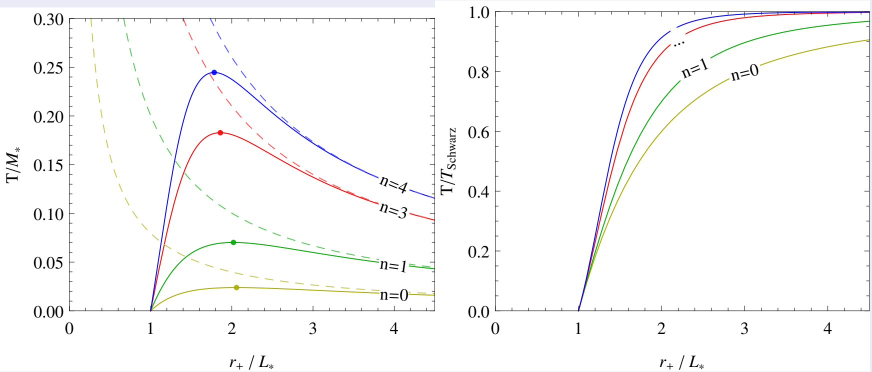
Thermodynamics

- BHs are significantly colder $\rightsquigarrow$ suppressed back reaction!

n	0	1	2	3	4	5	6	7
$r_{\max}/L_*$	2.06	1.60	1.48	1.41	1.36	1.33	1.30	1.28
$T_{\max}/M_*$	0.024	0.07	0.12	0.18	0.25	0.31	0.38	0.44

Table: Maximum temperatures $T_{\max} \equiv T(r_{\max})$ of the holographic black hole.

Temperatures in comparison with Schwarzschild-Tangherlini



Evaporation and grey body factors

- Particle spectrum

$$\frac{dN^{(s)}}{dt} = \sum_j \mathrm{N}_j \int \frac{g_{jn}^{(s)}(\omega)}{\exp\left(\hbar\omega/k_{\mathrm{B}}T\right) \pm 1} \frac{d\omega}{(2\pi)}$$

  where $g_{jk}^{(s)}$ is the grey body factor and N_j is the j-state multiplicity.
- Bulk/brane emission decreases with increasing n.
- e.g. for $n = 7$ bulk/brane 0.02% (in marked contrast to the usual result 93%)
- emission dominated by **soft particles mostly on the brane**.

  [PN & Winstanley JHEP **1111**, 075 (2011)]

Summary and conclusions

- Spacetime engineering is not aiming to a description at fundamental level

RATHER

- it aims to derive concrete phenomenological repercussions as a theoretical testbed for a theory;
- we offered some insights about microscopic black holes
- signatures might be surprising if we slightly deviate from the GR description.

Grazie per l'attenzione!

– Piero

Back up material I – Non local gravity action

We want to derive the holographic metric as a non-ocal action in place of the Einstein-Hilbert action:

$$I = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,G^{\mu\nu}\frac{\mathcal{A}(\Box/\mu^2)}{\Box}R_{\mu\nu}$$

To first approximation the non local action I leads to field equations

$$\mathcal{A}(\Box/\mu^2)\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)+\mathcal{O}(R_{\mu\nu}^2)=8\pi G T_{\mu\nu}$$

We want to shift all non local contributions to a smeared energy momentum tensor $\mathfrak{T}_{\mu\nu}=\mathcal{A}^{-1}(\Box/\mu^2)T_{\mu\nu}$ so

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G\mathfrak{T}_{\mu\nu}$$

Plan: Compute $\mathcal{A}(\Box/\mu^2)$.

Back up material II – Derivation of the nonlocal operator

We can determine the operator $\mathcal{A}^{-1}(\Box/\mu^2)$ in the Fourier space: Acting on the Schwarzschild source, it generates the smeared matter term.

$$\frac{m(r)}{M} = \left(\frac{1}{2\pi|\vec{x}|}\right)\frac{L_{\mathrm{P}}^2}{\left(|\vec{x}|^2 + L_{\mathrm{P}}^2\right)^2} = \mathcal{A}^{-1}\left(\Delta/M_P^2\right)\delta^3(\vec{x})$$

The result reads

$$\mathcal{A}^{-1}(p) = \frac{1}{2}\left[e^{p/\mu}\mathrm{E}_1(p/\mu)\left(\frac{1}{p/\mu} - 1\right) + e^{-p/\mu}\mathrm{Ei}(p/\mu)\left(\frac{1}{p/\mu} + 1\right)\right]$$
$$\approx 1 + \left[\frac{\gamma}{3} - \frac{4}{9} + \frac{1}{6}\ln\left(\frac{\Box}{\mu^2}\right)\right]\left(\frac{\Box}{\mu^2}\right) + \frac{1}{60}\left[3\gamma - 4 + \ln\left(\frac{\Box}{\mu^2}\right)\right]\left(\frac{\Box}{\mu^2}\right)^2$$
$$+ O(\Box^3/\mu^6)$$

Einstein Gravity is consistently recovered in the limit

$$\lim_{\mu\to\infty}\mathcal{A}\left(\Box/\mu^2\right) = 1.$$