Analog Hawking radiation in Bose-Einstein condensates

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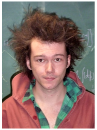
I. Carusotto

A. Recati



P.-É. Larré

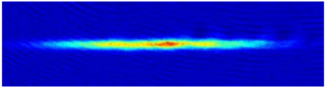
A. Kamchatnov

D. Boiron

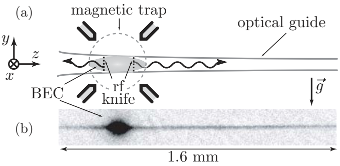
S. Fabbri

C. Westbrook

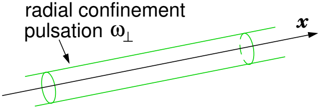
P. Zin



quasi-1D condensate
longitudinal size $\sim 10^2 \mu$m
transverse size $\sim 1\mu$m



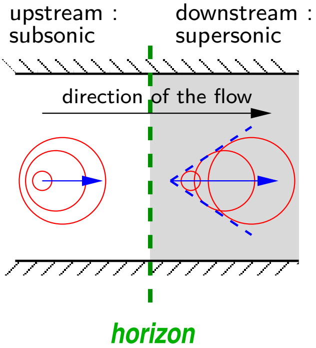
Guerin *et al.*, Phys. Rev. Lett. (2006)



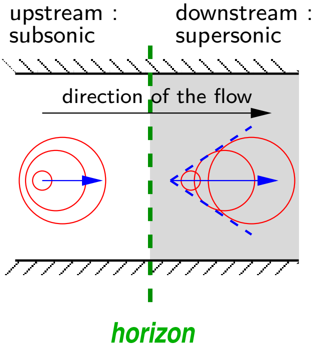
harmonic radial confinement :

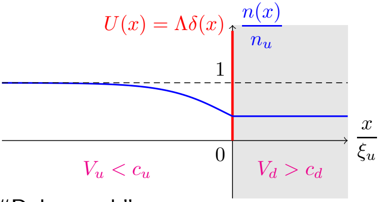
$$V_\perp(\vec{r}_\perp) = \frac{1}{2}\, m\, \omega_\perp^2\, r_\perp^2 \; .$$

$\rightarrow$ **1D model** : $\psi(x, t)$

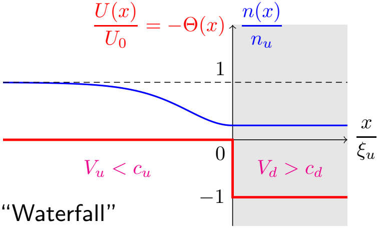

upstream :
subsonic
downstream :
supersonic
direction of the flow
horizon

Unruh, Phys. Rev. Lett. (1981)

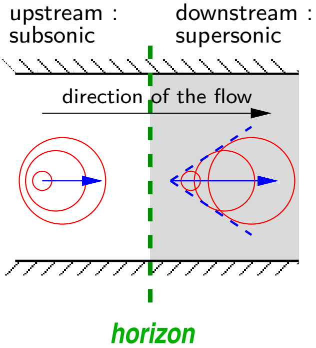


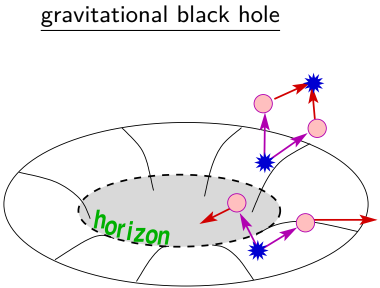


"Delta peak"



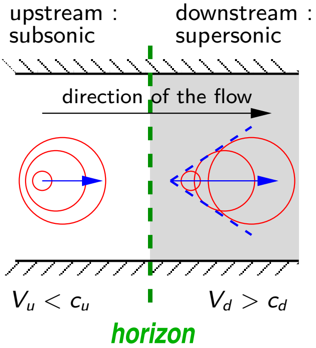
"Waterfall"

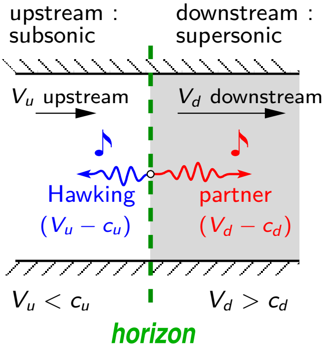


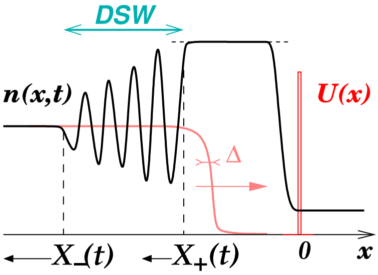


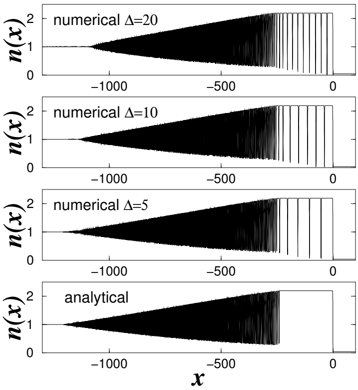
Hawking radiation 75'

Unruh, Phys. Rev. Lett. (1981)







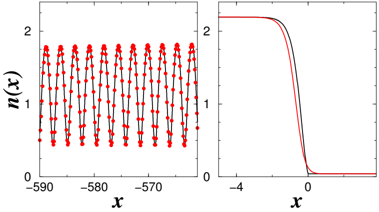


The profile in the region of the horizon only depends on v_u/c_u, but not of the initial profile. No hair theorem[a] ?

[a] F. Michel, R. Parentani, R. Zegers, Phys. Rev. D (2016)

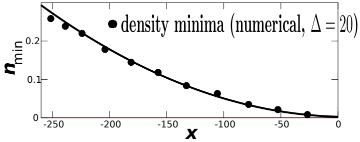
How to form a sonic horizon ?

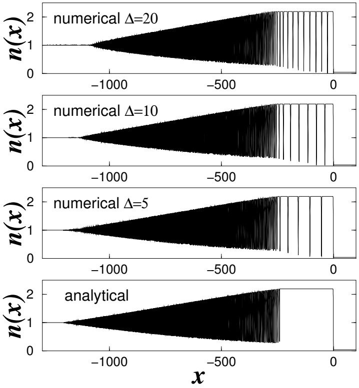
A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)



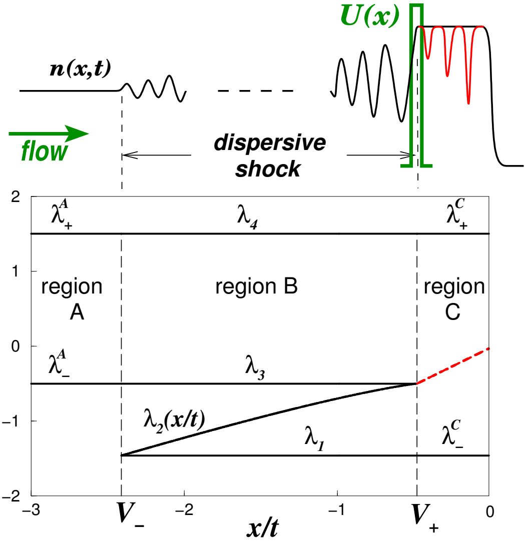
red=numerics

black=Whitham+transonic





Soliton train in the upstream plateau... experimental problem ?

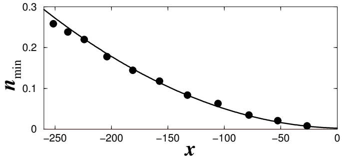


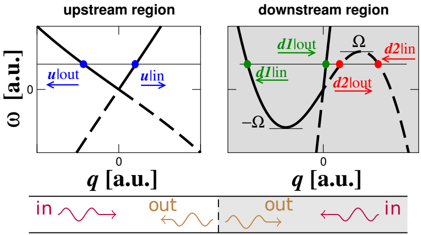
soliton train:

★ λ_1, λ_4: fixed
$\lambda_2(\frac{x}{t}) = \lambda_3(\frac{x}{t})$

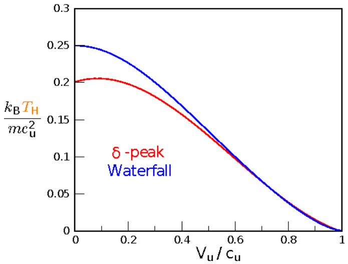
★ $\frac{x}{t} = \sum_{i=1}^{4} \lambda_i$
yields $\rightarrow \lambda_{2,3}\left(\frac{x}{t}\right)$

★ $n_{\min} = f(\lambda_i\,'\mathrm{s})$
$= f(\frac{x}{t})$

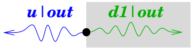




Hawking radiation in the $u|\text{out}$ channel.
Equivalent to a black body radiation of temperature $T_H \simeq 10\%\ \mu$

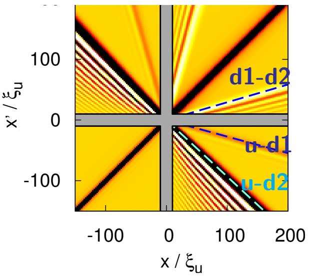


★ example of induced correlation:

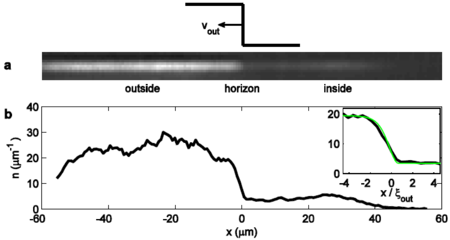


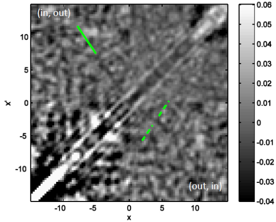
$x = (v_d + c_d)t$ correlates with
$x' = (v_u - c_u)t$

★ affects the density correlation pattern $\langle : n(x) n(x') : \rangle$



Larré *et al.*, Phys. Rev. A (2012)





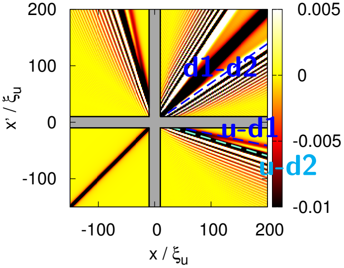
density profile near the horizon $\simeq$ waterfall

$n_u/n_d = 5.55$ 5.55

$c_u/c_d = 2.4$ 2.36

$V_u/c_u = 0.375$ 0.4245, $V_d/c_d = 3.25$ 5.55

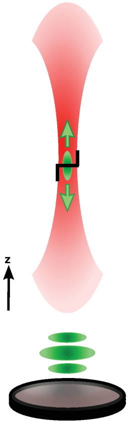
$T_H = 1.0$ nK $\quad T_H/\mu = 0.4$?

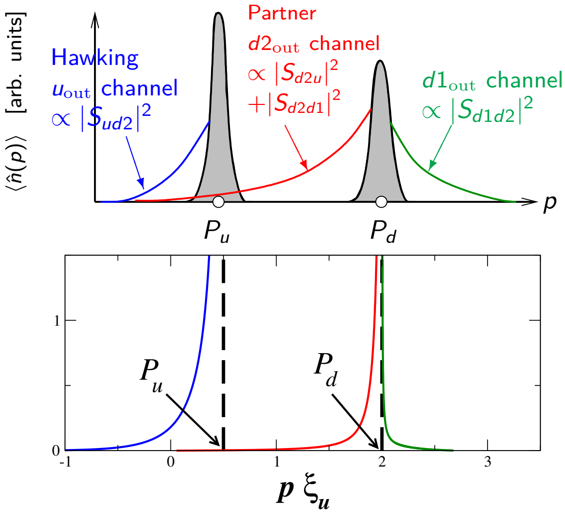


One body momentum distribution in the presence of a horizon

$T = 0$, adiabatic opening of the trap

Boiron *et al.* PRL (2015)

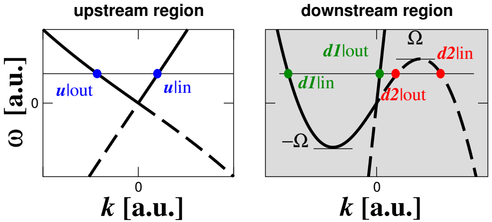




p, q : absolute momenta in units of ξ_u^{-1}

right plot: $g_2(p,q) \rightarrow$

where $g_2(p,q) = \dfrac{\langle : \hat{n}(p)\hat{n}(q) : \rangle}{\langle \hat{n}(p) \rangle \langle \hat{n}(q) \rangle}$

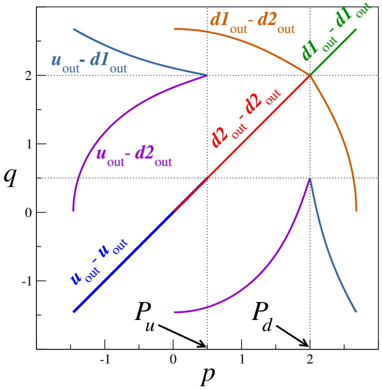


k : momentum relative to the condensate

$p = k + P_{(u/d)}$ where $P_{(u/d)} = mV_{(u/d)}$

$T = 0$ adiabatic opening

Boiron *et al.* PRL (2015)



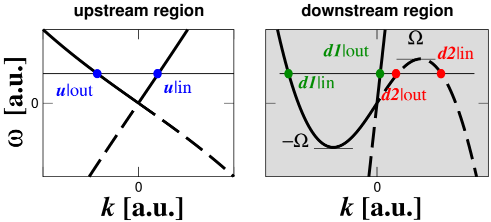
without horizon: $g_2 \equiv 1$

Two body momentum distribution in the presence of a horizon

p, q : absolute momenta in units of ξ_u^{-1}

right plot: $g_2(p,q) \to$

where $g_2(p,q) = \dfrac{\langle : \hat{n}(p)\hat{n}(q) : \rangle}{\langle \hat{n}(p)\rangle\langle \hat{n}(q)\rangle}$



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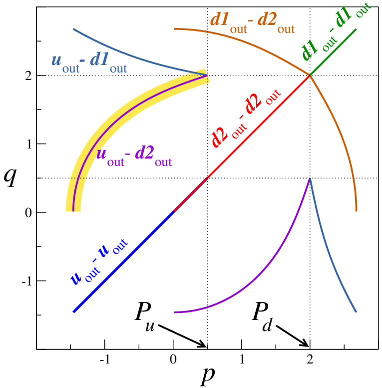
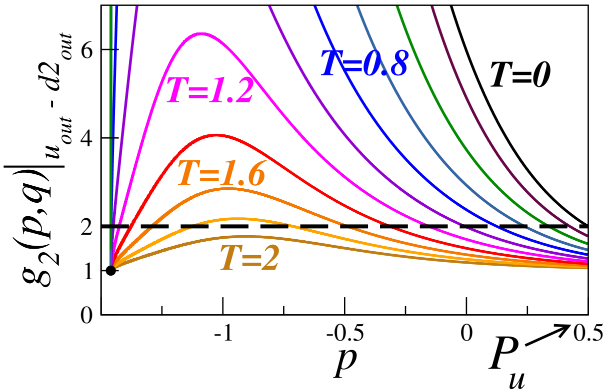


without horizon: $g_2 \equiv 1$

C.-S. violation : $$g_2(p,q)\Big|_{u_{\rm out}-d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2$$

Boiron *et al.* PRL (2015)



T in units of μ

$T_H = 0.13$

$V_u/c_u = 0.5$

$V_d/c_d = 4$

$V_d/V_u = 4$

$n_u/n_d = 4$

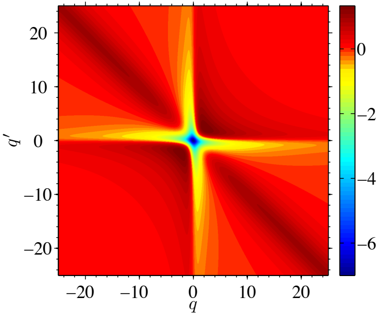
The NLS ↔ Gross-Pitaevskii eq. is a nonlinear **quantum field** eq. :

$$-\frac{\hbar^2}{2m}\partial_x^2\hat{\psi} + g\,\hat{\psi}^\dagger\hat{\psi}\,\hat{\psi} = i\,\hbar\,\partial_t\hat{\psi}\ ,\quad \text{with}\quad \left[\hat{\psi}(x,t),\hat{\psi}^\dagger(y,t)\right] = \delta(x-y)\ .$$

BEC : macroscopic occupation of the lowest quantum state: $\hat{\psi}(x,t) = \psi_{(0)}(x,t) + \hat{\phi}(x,t)$ (Bogoliubov 1947)

$\psi_{(0)}$	: solution of the (classical) NLS
$\hat{\phi}$	: solution of a linearized (quantum) eq.

makes it possible to consider vacuum fluctuations. In particular : **Hawking radiation** in a stationary, non uniform setting.

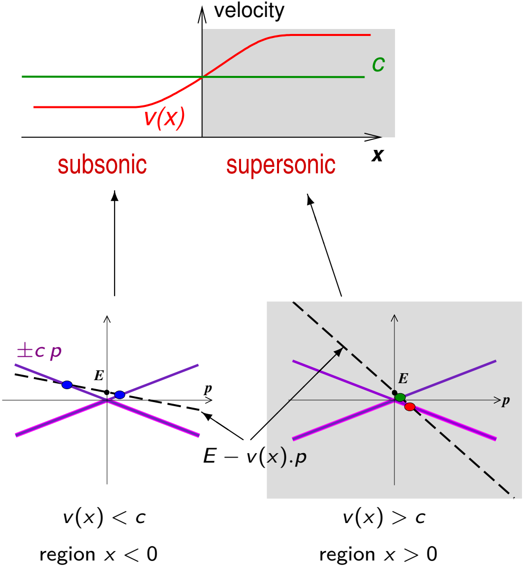


Mathey, Vishwanath, Altman, PRA (2009)

Bouchoule, Arzamasovs, Kheruntsyan, Gangardt, PRA (2012)

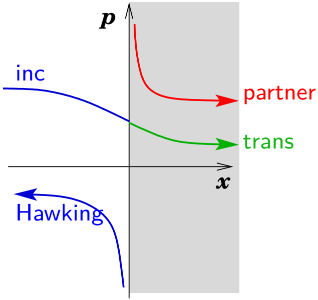
the position of the horizon is energy-dependent

model configuration :



$$E - v(x).p = \pm c\, p$$

phase space :



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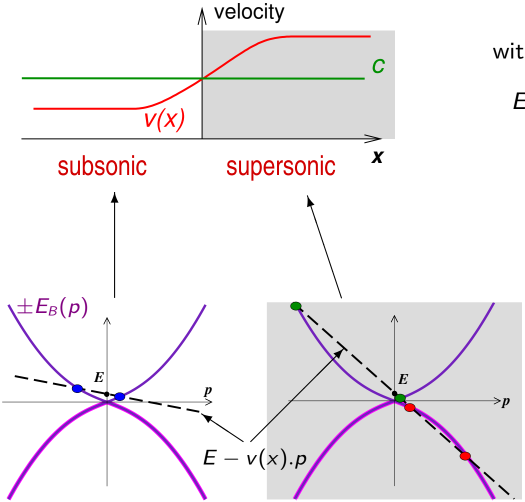


$v(x) < c$

region $x < 0$

$v(x) > c$

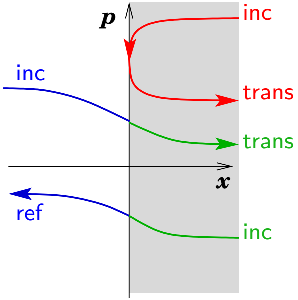
region $x > 0$

$$E - v(x).p = \pm\, E_B(p)$$

with

$$E_B(p) = c\, p\, \sqrt{1 + \xi^2 p^2/4}$$

phase space :



$$\hat{\phi}(x) = e^{iP_{(u/d)}x} \int_0^\infty \frac{d\omega}{\sqrt{2\pi}} \sum_{L\in\{U,D1\}} \left[u_L(x,\omega)\hat{a}_L(\omega) + v_L^*(x,\omega)\hat{a}_L^\dagger(\omega) \right]$$
$$+ e^{iP_{(u/d)}x} \int_0^\Omega \frac{d\omega}{\sqrt{2\pi}} \left[u_{D2}(x,\omega)\hat{a}_{D2}^\dagger(\omega) + v_{D2}^*(x,\omega)\hat{a}_{D2}(\omega) \right] .$$

- If $x \ll -\xi_u$: $u_U(x) = \mathcal{U}_{u|\mathrm{in}} e^{iq_{u|\mathrm{in}}x} + S_{uu}\mathcal{U}_{u|\mathrm{out}} e^{iq_{u|\mathrm{out}}x}$,
- If $x \gg \xi_d$: $u_U(x) = S_{d1,u}\mathcal{U}_{d1|\mathrm{out}} e^{iq_{d1|\mathrm{out}}x} + S_{d2,u}\mathcal{U}_{d2|\mathrm{out}} e^{iq_{d2|\mathrm{out}}x}$.

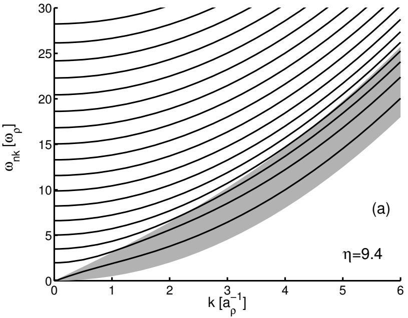
adiabatic opening of the trap: $\begin{pmatrix}\mathcal{U}(\omega)\\ \mathcal{V}(\omega)\end{pmatrix}_{\mathrm{mode}} \rightarrow \begin{pmatrix}1\\0\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$ for $d2|out$

see also: de Nova, Sols & Zapata PRA (2014)

- adiabaticity **always violated** for long wave-lengths, when $\omega \times t_{\mathrm{char}} \ll 1$

truly 1D ? No : transverse excitations

when $\hbar\omega_\perp \leq \mu$:



Zaremba, PRA (1998)
Stringari, PRA (1998)
Fedichev & Shlyapnikov, PRA (2001)
Tozzo & Dalfovo, PRA (2002)

modified dispersion relation :

$$\omega_0^2(q) = c_{1d}^2\, q^2 \left(1 - \tfrac{1}{48}(qR_\perp)^2 + \dots\right)$$

new channels :

$$\omega_{n\geq 1}^2(q) = 2n(n+1)\,\omega_\perp^2 + \tfrac{1}{4}(qR_\perp\omega_\perp)^2 + \dots$$

these new channels will be populated at $T = 0$

mass term $\neq$ Klein-Gordon

$\rightarrow$ new “in” modes

Conclusion

BECs offer interesting prospects to observe analogous Hawking radiation

[Steinhauer, arXiv:1510.00621]

general perspective : **quantum effects** with nonlinear matter waves

One- and two-body momentum distributions accessible by present day experimental techniques provide clear direct evidences

- ➪ of the occurrence of a sonic horizon.
- ➥ of the associated acoustic Hawking radiation.
- ☞ of the quantum nature of the Hawking process.
  - ☺ The signature of the quantum behavior persists even at temperatures larger than the chemical potential.

$\langle \ldots \rangle \overset{def}{=} \mathrm{Tr}(\rho \ldots)$

$$\begin{aligned}\langle \hat{n}^2 \rangle &= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{a}^\dagger 1\, \hat{a} \rangle \\ &= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle + \langle \hat{n} \rangle\end{aligned}$$

$$\begin{aligned}0 \leq \langle \delta n^2 \rangle &\overset{def}{=} \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \\ &= \underbrace{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle - \langle \hat{n} \rangle^2}_{sign?} + \langle \hat{n} \rangle\end{aligned}$$

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Cauchy-Schwarz: $|\langle \hat{A} \rangle|^2 \leq \langle \hat{A}^\dagger \hat{A} \rangle$

Hence	$|\langle \hat{a}\hat{a} \rangle|^2 \leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$
But	$|\langle \hat{a}^\dagger \hat{a} \rangle|^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$

$$\langle ... \rangle \overset{def}{=} \mathrm{Tr}(\rho ...)$$

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But	$|\langle \hat{a}^\dagger \hat{a} \rangle|^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$

stupid theoretical example

average over a number state: $\rho \equiv |n\rangle\langle n|$

$$\begin{aligned}\langle \hat{a}\hat{a} \rangle^2 &= 0 \\ \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle &= n(n-1) \\ \langle \hat{a}^\dagger \hat{a} \rangle^2 &= n^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle\end{aligned}$$

a number state is clearly sub-Poissonian !

$\langle \ldots \rangle \stackrel{def}{=} \mathrm{Tr}(\rho \ldots)$

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Cauchy-Schwarz: $|\langle \hat{A} \rangle|^2 \leq \langle \hat{A}^\dagger \hat{A} \rangle$

Hence	$|\langle \hat{a}\hat{a} \rangle|^2 \leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$
But	$|\langle \hat{a}^\dagger \hat{a} \rangle|^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle$

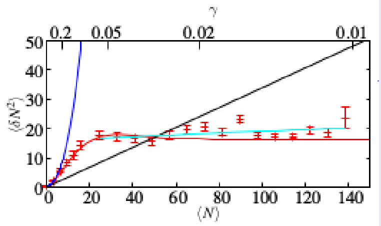
stupid theoretical example

average over a number state: $\rho \equiv |n\rangle\langle n|$

$$\begin{aligned} \langle \hat{a}\hat{a} \rangle^2 &= 0 \\ \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle &= n(n-1) \\ \langle \hat{a}^\dagger \hat{a} \rangle^2 &= n^2 \not\leq \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a} \rangle \end{aligned}$$

a number state is clearly sub-Poissonian !

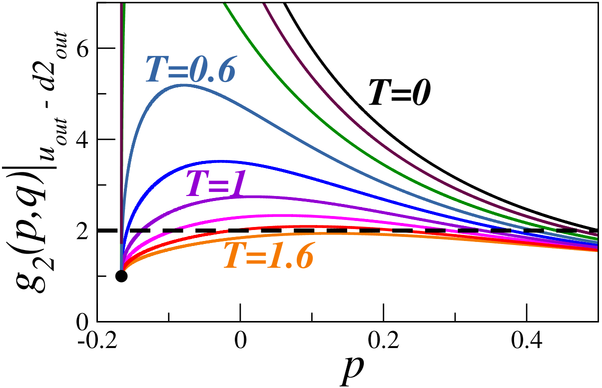
experimental results



Poissonian limit : $\langle \delta N^2 \rangle = 0.34 \langle N \rangle$
Ideal Bose gas
Yang-Yang
Quasi-cond.

Jacqmin *et al.*, *PRL (2011)*

$$g_2(p,q)\Big|_{u_{\rm out}-d2_{\rm out}} > \sqrt{g_2(p,p)\Big|_{u_{\rm out}} \times\, g_2(q,q)\Big|_{d2_{\rm out}}} \equiv 2\ ,$$



T in units of μ

$T_H = 0.12$

$V_u/c_u = 0.5$

$V_d/c_d = 1.83$

$V_d/V_u = 2.34$

$n_u/n_d = 2.37$