Gravitational collapse in the AS scenario: Kuroda-Papapetrou RG-improved model

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Gravitational collapse and singularities

Penrose and Hawking

Under reasonable conditions, collapse became inevatable once a *trapped surface* forms.

- The collapse give rise to a **gravitational singularity**
- Is the singularity *always* covered by an **event horizon**?
- Can **naked singularities** arise from gravitational collapse?

Cosmic Censorship Conjecture (CCC) - Penrose (1969)

The generic singularities arising in the gravitational collapse are always covered by an event horizon.

Gravitational collapse: Vaidya space-time

The metric of a spherically symmetric collapsing object is:

$$ds^2 = -f(r,v)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2 \tag{1}$$

Where $f(r,v)$ is given by:

$$f(r,v) = 1 - \frac{2\,G_0\,m(v)}{r} \tag{2}$$

The **mass function** $m(v)$ depends on the advanced time v.

In the **Kuroda-Papapetrou model** the mass function is:

$$m(v) = \begin{cases} 0 & v < 0 \\ \lambda v & 0 \leq v < \bar{v} \\ \bar{m} & v \geq \bar{v} \end{cases} \tag{3}$$

In this model, for $\lambda \leq \frac{1}{16\, G_0}$, *a far away observer see a persistent naked singularity*.

Y. Kuroda, Prog. Theor. Phys. 72, 63 (1974)

A. Papapetrou, *A random walk in relativity and cosmology*. Hindustan Publishing Co., New Delhi, India (1985)

Gravitational collapse in the generalized Vaidya space-time

Mkenyeleye, Goswami, Maharaj. Phys. Rev. D 90, 064034 (2014)

- Which is the mathematical condition to have a naked singularity?
- Which is its strength?

$$ds^2 = -f(r,v)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2 \tag{4}$$

$$f(r,v) = 1 - \frac{2\,M(r,v)}{r} \tag{5}$$

We have to study the geodesic equation:

$$\frac{\mathrm{d}v}{\mathrm{d}r} = 2\left(1 - \frac{2\,M(r,v)}{r}\right)^{-1} \tag{6}$$

It is useful to write the geodesic equation as the following dynamical system:

$$\begin{cases} \frac{\mathrm{d}v(t)}{\mathrm{d}t} = N(r,v) = 2\,r \\ \frac{\mathrm{d}r(t)}{\mathrm{d}t} = D(r,v) = r - 2\,M(r,v) \end{cases} \tag{7}$$

Eigenvalues of the stability matrix J:

$$\chi_{\pm} = \frac{1}{2}\left(\mathrm{Tr}J \pm \sqrt{(\mathrm{Tr}J)^2 - 4\,\det J}\right) \tag{8}$$

- $(\mathrm{Tr}J)^2 - 4\det J \geq 0 \quad \det J > 0 \qquad \Rightarrow$ **Node** (naked singularity)
- $(\mathrm{Tr}J)^2 - 4\det J < 0 \qquad \Rightarrow$ **Spiral node** (black hole)

The singularity "strength"

A singularity is said to be **strong** if an object falling into the singularity is destroyed by the gravitational tidal forces. Otherwise it is called weak or **integrable** ($\Rightarrow$ the space-time is **extendable**).

The strength of the singularity is given by:

$$S = \frac{\dot{M}_{FP} X_{FP}^2}{2} \qquad X_{FP} \equiv \lim_{(r,v)\to \mathrm{FP}} \frac{v(r)}{r} \tag{9}$$

The singularity is *strong* if $S > 0$, viceversa it is *integrable*.

Mkenyeleye, Goswami, Maharaj. Phys. Rev. D 90, 064034 (2014)
Strokov, Lukash, Mikheeva. Int. J. Mod. Phys. A 31, 1641018 (2016)

Kuroda-Papapetrou RG-improved model

The classical Vaidya space-time is: $ds^2 = -f_c(r,v)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$

$$f_c(r,v) = 1 - \frac{2\,m(v)\,G_0}{r} \tag{10}$$

Asymptotic Safety and running Newton constant

By using the exact RG:

$$G(k) = \frac{G_0}{1 + \omega\,G_0\,k^2} \tag{11}$$

Where $\omega = 1/g_*$ and k is the infrared cutoff scale.

M. Reuter, Phys. Rev. D 57, 971 (1998)

Metric improvement

The idea is to study the gravitational collapse arising from the **RG-improved Vaidya metric**:

$$f_c(r,v) \;\longrightarrow\; f_q(r,v) = 1 - \frac{2\,m(v)}{r}\,\frac{G_0}{1+\omega\,G_0\,[k(r)]^2} \tag{12}$$

The question is: Which is, in this case, the correct cutoff identification?

- The best choice is to relate $k(r)$ with the energy density of a null free falling observer:

$$k(r) \equiv \xi\,\sqrt[4]{\rho(r,v)} = \xi\,\sqrt[4]{\frac{\dot{m}(v)}{4\pi r^2}} \tag{13}$$

Babic, Guberina, Horvat, Stefancic. Phys.Rev. D71 (2005) 124041
Bonanno, Esposito, Rubano, Scudellaro. Class. Quant. Grav. 23 (2006) 3103

By using this cutoff identification, and assuming $m(v) = \lambda\, v$:

$$f_{\mathrm{q}}(r, v) = 1 - \frac{2\,\lambda\, G_0\, v}{r + \alpha\,\sqrt{\lambda}} \qquad \alpha = \frac{\xi^2\, G_0}{\sqrt{4\pi}\, g_*} \tag{14}$$

Compare:

$$f_{\mathrm{c}}(r, v) = 1 - \frac{2\,\lambda\, G_0\, v}{r} \tag{15}$$

The effect of a **running Newton constant** is to produce a shift to r(v):

$$r(v) \longrightarrow r(v) + \alpha\sqrt{\lambda} \tag{16}$$

General solution for the outgoing radial null geodesics

Exact analytic solutions:

$$\mathrm{Log}\left[2\lambda\, G_0 v^2 - (r(v) + \alpha\sqrt{\lambda})v + 2\,(r(v) + \alpha\sqrt{\lambda})^2\right] +$$

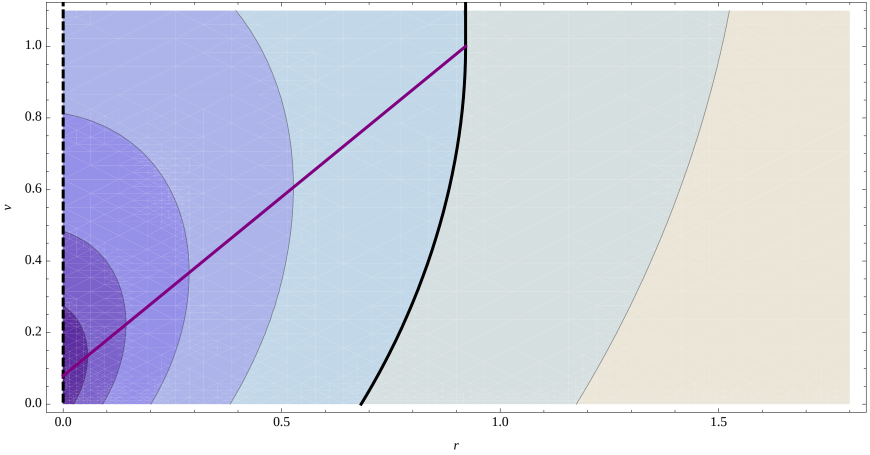
$$+\frac{-2\,\mathrm{ArcTan}\left[\frac{v-4\,[\,r(v)+\alpha\sqrt{\lambda}\,]}{v\sqrt{-1+16\,\lambda\,G_0}}\right]}{\sqrt{-1+16\,\lambda\,G_0}} = C \qquad (17)$$

Observations:

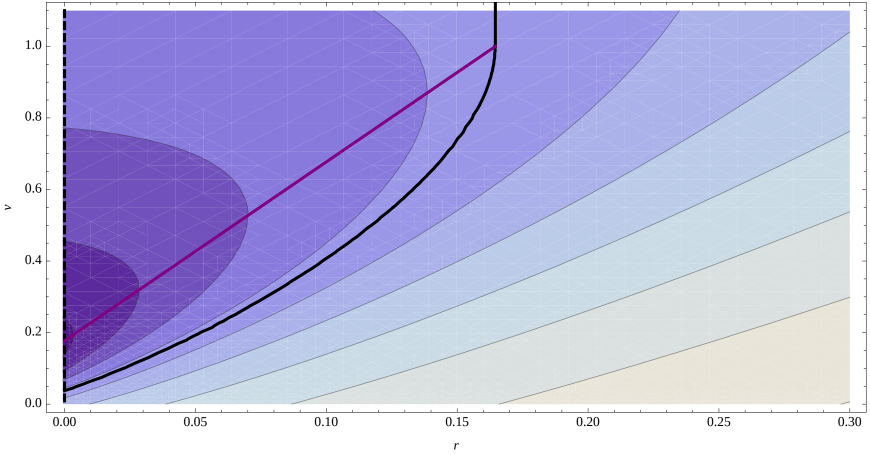
- The "critical value" is now $\lambda_c > \frac{1}{16\,G_0}$
- The improved geodesic equation admit the constant solutions:

$$r_{\pm}(v) = -\alpha\sqrt{\lambda} + \mu_{\pm}\, v \qquad \mu_{\pm} = \frac{1}{4}\left(1 \pm \sqrt{1 - 16\,\lambda\,G_0}\right)$$

$\lambda > \lambda_c$ The singularity is behind the horizon: BH



$\lambda \leq \lambda_c$ Naked singularity



On the nature of the singularity

Eingeinvalues of the **stability matrix** J: $\chi_{\pm} = \frac{1}{2}\left(\mathrm{Tr}J \pm \sqrt{(\mathrm{Tr}J)^2 - 4\,\mathrm{det}J}\right)$

Classical Kuroda-Papapetrou model:

- $\mathrm{Tr}J = 1 \quad \mathrm{det}J = 4\,\lambda\,G_0 \quad \Rightarrow \quad \chi_{\pm} = \frac{1}{2}\left(1 \pm \sqrt{1 - 16\,\lambda\,G_0}\right)$
- The origin $(0,0)$ is a **naked singularity** if $\lambda \leq \frac{1}{16\,G_0}$, $S > 0$

Improved Kuroda-Papapetrou model:

- $\mathrm{Tr}J = 1 - \frac{2\lambda v_0\,G_0}{\alpha\sqrt{\lambda}} \qquad \mathrm{det}J \propto G(r)]_{r\to 0} = 0$ **Fixed Points line**
- Strength: $S \propto G(0) = 0$ **Integrable!**
- There is no dependence on the critical value λ_c

Non-linearity effects

Look again at the **full improved geodesic equation**, written as:

$$\begin{cases} \frac{\mathrm{d}v(t)}{\mathrm{d}t} = 2\, r(t) \\ \frac{\mathrm{d}r(t)}{\mathrm{d}t} = r(t) - 2\,\lambda\, G_0\, v(t)\, \frac{r(t)}{r(t)+\alpha\sqrt{\lambda}} \end{cases} \tag{18}$$

Where:

$$\alpha = \frac{\xi^2\, G_0}{\sqrt{4\pi}\, g_*} \propto M_{\mathrm{pl}}^{-2} \tag{19}$$

Region far from the singularity $r = 0$

$$r \gg \alpha\sqrt{\lambda} \qquad \Leftrightarrow \qquad [k(r)]^2 \ll M_{\mathrm{pl}}^2 \qquad \textbf{classical region}$$

We found that **the NL effects (near classical region) restore the λ_c-dependence**.

Conclusions

- We studied a **RG-improved Kuroda-Papapetrou model**;
- We found that the only effect of a **running Newton constant** is to turn a strong naked singularity into a line of integrable singularities;
- The space-time is then extandable beyond $r = 0$, but the Cosmic Censorship Hypothesis is violated;
- The presence of the limiting value λ_c is a **purely classical effect**: the formation of naked singularities in the KP model is due to the gravitational collapse dynamics in the classical region.

Thanks for your attention

Which is the behavior of the trajectories near the FP line?

For a given fixed point $(0, v_0)$, the characteristic directions are:

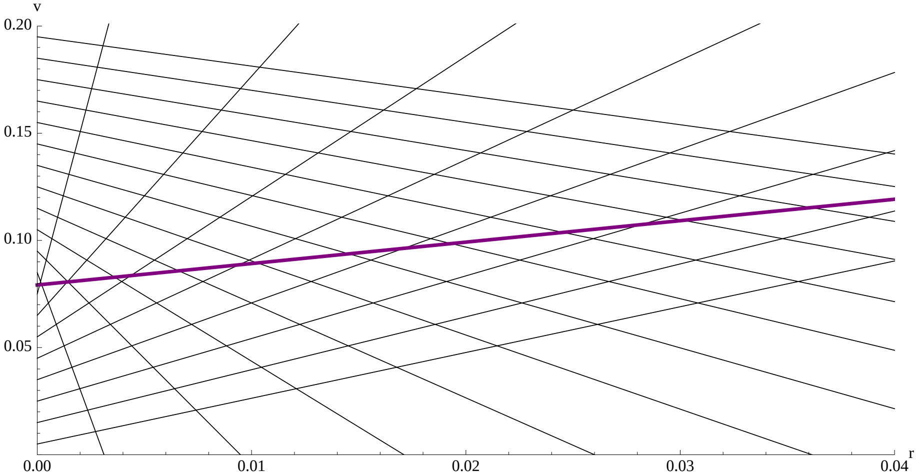
$$r = 0 \qquad \longleftrightarrow \qquad \chi_- = 0 \qquad \text{(marginal)} \tag{20}$$

$$v = v_0 + \frac{2\,r}{\chi_+(v_0)} \qquad \longleftrightarrow \qquad \chi_+(v_0) \equiv \mathrm{Tr}J = 1 - \frac{2\lambda v_0\, G_0}{\alpha\,\sqrt{\lambda}} \tag{21}$$

Non-marginal direction:

- **positive slope** $\frac{2}{\chi_+(v_0)} > 0$ $\Leftrightarrow$ **Repulsive direction**
- **negative slope** $\frac{2}{\chi_+(v_0)} < 0$ $\Leftrightarrow$ **Attractive direction**
- **inversion point** $\bar{v}_0 = \frac{\alpha\sqrt{\lambda}}{2\,\lambda\,G_0}$ $\Leftrightarrow$ **Apparent horizon**

Characteristic directions “phase diagram”



- For example, if $\lambda > \lambda_c$