Unparticle Parameters in the Light of the Hydrogen Ground State

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2nd FLAG Meeting "The Quantum and Gravity"

Trento, June 6, 2016

M. F. Wondrak, P. Nicolini, M. Bleicher, arXiv:1603.03319 [hep-ph]

Unparticle Physics Constraints from the Hydrogen Atom

Outline

- Overview of Unparticle Physics
- Static Unparticle Potential
- Unparticle-Corrected Hydrogen Atom
- Constraints Using the Hydrogen Atom

Overview of Unparticle Physics

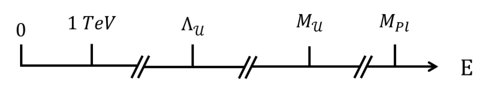
Unparticle Physics

- Suggested by Howard Georgi in 2007.
- Massive stuff exhibiting scale invariance.
- *Un*like normal *particles*, e.g. Standard Model particles.
- Features:
  - Appearance as a non-integral number of invisible particles.
  - Hardly interacting with ordinary particles $\rightarrow$ not yet observed.

H. Georgi, Phys. Rev. Lett. **98** (2007) 221601; H. Georgi, Phys. Lett. **B650** (2007) 275

Overview of Unparticle Physics

Energy Scales



- Very high energies:
  Standard Model sector, Banks-Zaks sector, messenger field.
- Below the mass scale of the messenger field, $M_\mathcal{U}$:

$$\mathcal{L}_{\text{int}} = \frac{1}{M_\mathcal{U}^k} \mathcal{O}_{\text{SM}} \mathcal{O}_{\mathcal{BZ}}$$

  with $k = d_{\text{SM}} + d_{\mathcal{BZ}} - 4$.
- Below the energy scale $\Lambda_\mathcal{U}$ at which scale invariance evolves:

$$\mathcal{L}_{\text{int}} = \frac{C_\mathcal{U} \Lambda_\mathcal{U}^{d_{\mathcal{BZ}} - d_\mathcal{U}}}{M_\mathcal{U}^k} \mathcal{O}_{\text{SM}} \mathcal{O}_\mathcal{U} = \frac{\lambda}{\Lambda_\mathcal{U}^{d_{\text{SM}} + d_\mathcal{U} - 4}} \mathcal{O}_{\text{SM}} \mathcal{O}_\mathcal{U}$$

  with unparticle scaling dimension $d_\mathcal{U}$ and unparticle charge $\lambda = C_\mathcal{U} \left(\frac{\Lambda_\mathcal{U}}{M_\mathcal{U}}\right)^k < 1$.

Overview of Unparticle Physics

Unparticle Propagator

- The propagator for a scalar unparticle field is determined by scale invariance:

$$\tilde{\Delta}_{\mathrm{F}}(p) = \frac{A_{d_{\mathcal{U}}}}{2\sin\left(d_{\mathcal{U}}\pi\right)} \frac{1}{\left(-p^2 - i\epsilon\right)^{2-d_{\mathcal{U}}}}$$

$$1 < d_{\mathcal{U}} < 2$$

- Normalization factor:

$$A_{d_{\mathcal{U}}} = \frac{16\,\pi^{\frac{5}{2}}}{\left(2\pi\right)^{2d_{\mathcal{U}}}} \frac{\Gamma\left(d_{\mathcal{U}} + \frac{1}{2}\right)}{\Gamma(d_{\mathcal{U}} - 1)\,\Gamma(2d_{\mathcal{U}})}$$

- Effective Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\,\partial_\mu\phi\,\frac{2\sin(d_{\mathcal{U}}\pi)}{A_{d_{\mathcal{U}}}}\left(\partial^\nu\partial_\nu\right)^{1-d_{\mathcal{U}}}\,\partial^\mu\phi$$

P. Gaete and E. Spallucci, Phys. Lett. **B661** (2008) 319–324;
N. V. Krasnikov, Int. J. Mod. Phys. **A22** (2007) 5117–5120

Overview of Unparticle Physics

Gravitational Aspects

- Generalized Einstein-Hilbert action:

$$S_{\mathcal{U}} = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{g} \left[1 + \frac{A_{d_{\mathcal{U}}}}{(2d_{\mathcal{U}}-1)\sin(\pi d_{\mathcal{U}})} \left(\frac{\kappa_*}{\kappa}\right)^2 \left(\frac{-\Box}{\Lambda_{\mathcal{U}}^2}\right)^{1-d_{\mathcal{U}}} \right]^{-1} R$$

- Spherically symmetric, static black hole of mass M:

$$\mathrm{d}s^2 = g_{00}\,\mathrm{d}t^2 - \frac{1}{g_{00}}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

where

$$g_{00} = -1 + \frac{2G_{\mathrm{N}}M}{r} \left[1 + \frac{1}{\pi^{2d_{\mathcal{U}}-1}} \frac{\Gamma\left(d_{\mathcal{U}} - \frac{1}{2}\right)\Gamma\left(d_{\mathcal{U}} + \frac{1}{2}\right)}{\Gamma(2d_{\mathcal{U}})} \left(\frac{\kappa_*}{\kappa}\right)^2 \frac{1}{(\Lambda_{\mathcal{U}} r)^{2d_{\mathcal{U}}-2}} \right]$$

- Unparticle contribution to the potential in the weak-field limit:

$$V_{\mathcal{U}} \propto \frac{1}{r^{2d_{\mathcal{U}}-1}}$$

J. R. Mureika, Phys. Lett. **B660** (2008) 561–566; P. Gaete and E. Spallucci, Phys. Lett. **B661** (2008) 319–324; P. Gaete, J. A. Helayël-Neto and E. Spallucci, Phys. Lett. **B693** (2010) 155–158

Static Unparticle Potential

Static Unparticle Potential

- Free field coupled to external sources:

$$V = -\lim_{T\to\infty} \frac{W[J]|_{\mathrm{nsi}}}{T}$$

where $W[J]|_{\mathrm{nsi}}$ is the generating functional of connected diagrams without self-interactions.

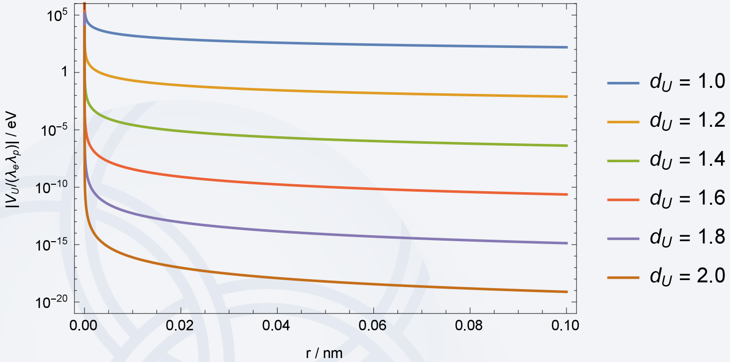
- Unparticle interaction energy $V_{\mathcal{U}}$ between electron and proton in the static case:

$$V_{\mathcal{U}} = -\frac{\sqrt{\pi}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma\left(d_{\mathcal{U}} - \frac{1}{2}\right)}{\Gamma(d_{\mathcal{U}})} \frac{\lambda_{\mathrm{e}}\, \lambda_{\mathrm{p}}}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}\, r^{2d_{\mathcal{U}}-1}}$$

P. Gaete and E. Spallucci, Phys. Lett. **B661** (2008) 319–324

Static Unparticle Potential

Visualization of the Unparticle Potential for $\Lambda_{\mathcal{U}} = 10$ TeV

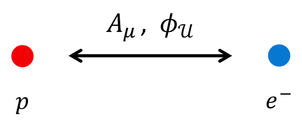


$$\frac{V_{\mathcal{U}}}{\lambda_{\rm e}\,\lambda_{\rm p}} = -\frac{\sqrt{\pi}}{(2\pi)^{2d_{\mathcal{U}}}}\,\frac{\Gamma\left(d_{\mathcal{U}}-\frac{1}{2}\right)}{\Gamma(d_{\mathcal{U}})}\,\frac{1}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}\,r^{2d_{\mathcal{U}}-1}}$$

Unparticle-Corrected Hydrogen Atom

Approach

- Extend the interaction between electron and proton by unparticle exchange:



- Total potential of the hydrogen atom:

$$\begin{aligned} V_{\text{tot}}(r) &= V_{\text{C}}(r) + V_\mathcal{U}(r) \\ &= -\frac{e^2}{r} - \underbrace{\frac{\sqrt{\pi}}{(2\pi)^{2d_\mathcal{U}}} \frac{\Gamma\left(d_\mathcal{U} - \frac{1}{2}\right)}{\Gamma(d_\mathcal{U})}}_{\xi_{d_\mathcal{U}}} \frac{\lambda_{\text{e}}\,\lambda_{\text{p}}}{\Lambda_\mathcal{U}^{2d_\mathcal{U}-2}\, r^{2d_\mathcal{U}-1}} \end{aligned}$$

- Solutions for a spherically symmetric potential:

$$\psi_{nlm}(r, \vartheta, \varphi) = \frac{u_{nl}(r)}{r}\, Y_{lm}(\vartheta, \varphi)$$

Unparticle-Corrected Hydrogen Atom

Schrödinger Description

- Radial Schrödinger equation including virtual unparticle exchange:

$$\left(-\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{2\mu}\frac{l\,(l+1)}{r^2}-\frac{e^2}{r}-\xi_{d_{\mathcal{U}}}\frac{\lambda_{\mathrm{e}}\,\lambda_{\mathrm{p}}}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}\,r^{2d_{\mathcal{U}}-1}}\right)u_{nl}(r)=E_{nlm}\,u_{nl}(r)$$

- Hydrogen ground state wave function:

$$\psi_{100}(r,\,\vartheta,\,\varphi)=\frac{2\,a^{-\frac{3}{2}}}{\sqrt{4\pi}}\,e^{-\frac{r}{a}}$$

with $\mu := \frac{m_{\mathrm{e}}\,m_{\mathrm{p}}}{m_{\mathrm{e}}+m_{\mathrm{p}}}$ and $a=\frac{\hbar^2}{\mu\,e^2}=\frac{1}{\alpha\,\mu}$.

- First order energy shift of the ground state:

$$\begin{aligned}\Delta_{100}^{(1)} &= \left\langle 100^{(0)}\,\middle|\,V_{\mathcal{U}}\,\middle|\,100^{(0)}\right\rangle \\ &= -\frac{1}{(2\pi)^{2d_{\mathcal{U}}-2}}\,\frac{(d_{\mathcal{U}}-1)\,\left(\frac{3}{2}-d_{\mathcal{U}}\right)}{\sin(2d_{\mathcal{U}}\pi)\,\left(\Gamma(d_{\mathcal{U}})\right)^2}\,\frac{\lambda_{\mathrm{e}}\,\lambda_{\mathrm{p}}}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}\,a^{2d_{\mathcal{U}}-1}}\end{aligned}$$

Unparticle-Corrected Hydrogen Atom

Hydrogen Atom Ground State

Energy	Description	Value
$E_{\mathrm{th}}^{\mathrm{S}}$	Schrödinger, non-relativistic	$-13.598\,286\,\mathrm{eV}$
$E_{\mathrm{th}}^{\mathrm{S,\,rel}}$	Schrödinger, incl. fine-structure	$-13.598\,467\,\mathrm{eV}$
$E_{\mathrm{th}}^{\mathrm{D}}$	Dirac	$-13.598\,467\,\mathrm{eV}$
$E_{\mathrm{th}}^{\mathrm{QED}}$	currently best theor. value	$-13.598\,434\,49\ (9)\ \mathrm{eV}$ $3\,288\,086\,857.127\,6\ (3\,1)\ \mathrm{MHz}\cdot h$
E_{exp}	currently best exp. value	$-13.598\,434\,48\ (9)\ \mathrm{eV}$ $3\,288\,086\,856.8\ (0.7)\ \mathrm{MHz}\cdot h$

A. E. Kramida, Atomic Data and Nuclear Data Tables **96** (2010) 586–644
U. D. Jentschura et al., http://physics.nist.gov/HDEL [accessed 11 January 2016]

Constraints Using the Hydrogen Atom

Experimental Bounds

- Condition:

$$\begin{aligned}\left|E_{\mathrm{th},\mathcal{U}} - E_{\mathrm{th}}\right| &< \delta E_{\mathrm{exp}} \\ \left|E^{\mathrm{S}}_{\mathrm{th},\mathcal{U}} - E^{\mathrm{S}}_{\mathrm{th}}\right| \simeq \left|\Delta^{(1)}_{100}\right| &< \delta E_{\mathrm{th}} + \delta E_{\mathrm{exp}} \\ \left|\frac{\Delta^{(1)}_{100}}{E^{\mathrm{S}}_{\mathrm{th}}}\right| &< \frac{\delta E_{\mathrm{th}} + \delta E_{\mathrm{exp}}}{\left|E^{\mathrm{S}}_{\mathrm{th}}\right|} \equiv \delta_{\mathrm{max}}\end{aligned}$$

- Conservative Approach:

$$\frac{\delta E_{\mathrm{th}}}{\left|E^{\mathrm{S}}_{\mathrm{th}}\right|} \simeq \left|\frac{E^{\mathrm{QED}}_{\mathrm{th}} - E^{\mathrm{S}}_{\mathrm{th}}}{E^{\mathrm{S}}_{\mathrm{th}}}\right| = 1.1 \times 10^{-5} \to \delta_{\mathrm{max}}$$

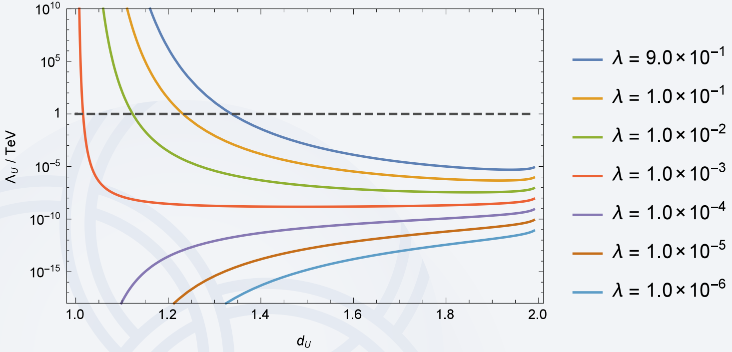
$$\left|\frac{\Delta^{(1)}_{100}}{E^{\mathrm{S}}_{\mathrm{th}}}\right| = \frac{2\alpha^{2d_{\mathcal{U}}-3}\,\mu^{2d_{\mathcal{U}}-2}}{(2\pi)^{2d_{\mathcal{U}}-2}} \frac{(d_{\mathcal{U}}-1)\left(\frac{3}{2}-d_{\mathcal{U}}\right)}{\sin(2\pi d_{\mathcal{U}})\,(\Gamma(d_{\mathcal{U}}))^2} \frac{|\lambda_{\mathrm{e}}\,\lambda_{\mathrm{p}}|}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}}$$

- Lower bound for $\Lambda_{\mathcal{U}}$:

$$\Lambda_{\mathcal{U}} \geq \frac{\alpha\,\mu}{2\pi}\left(\frac{2}{\alpha}\,\frac{(d_{\mathcal{U}}-1)\left(\frac{3}{2}-d_{\mathcal{U}}\right)}{\sin(2\pi d_{\mathcal{U}})\,(\Gamma(d_{\mathcal{U}}))^2}\,\frac{|\lambda_{\mathrm{e}}\,\lambda_{\mathrm{p}}|}{\delta_{\mathrm{max}}}\right)^{\frac{1}{2d_{\mathcal{U}}-2}}$$

Constraints Using the Hydrogen Atom

Visualization of the Constraint



Assuming $|\lambda_{\mathrm{e}}| = |\lambda_{\mathrm{p}}| \equiv \lambda$:

$$\Lambda_{\mathcal{U}} \geq \frac{\alpha\,\mu}{2\pi}\left(\frac{2}{\alpha}\,\frac{(d_{\mathcal{U}}-1)\left(\frac{3}{2}-d_{\mathcal{U}}\right)}{\sin(2\pi d_{\mathcal{U}})\,(\Gamma(d_{\mathcal{U}}))^2}\,\frac{\lambda^2}{\delta_{\max}}\right)^{\frac{1}{2d_{\mathcal{U}}-2}}$$

Unparticle Physics Constraints from the Hydrogen Atom

Conclusion

- Unparticle stuff is an exciting extension of the Standard Model showing the effects of scale invariance.
- The coupling to ordinary matter can be studied in order to constrain the allowed parameter space of the theory.
- The hydrogen atom is a well investigated system — in particular its ground state energy — and thus leads to stringent bounds on the unparticle parameter space for $d_{\mathcal{U}} < 1.4$.
- Unparticle effects might be tested in the realm of atomic physics!